

May 20

My dear ~~Raymond~~

I see that in
putting up my letter
to you last night

I must have put in
the wrong first sheet

Please substitute the
enclosed

T.O. Milford the 1905 Aug 19

My dear Professor Bogue:

I received today your highly important Memoir, and although I have not yet had time to read far into Chapter II, I will venture on a few remarks which may for aught I know be contained in the Memoir itself.

It appears that $O(\alpha)$ expresses a relation among the "elements" of α , and although it does not seem to be involved in the definition L819, you state that this does not depend on the order of the "elements". It is therefore a relation of equiparance. The negative of $O(\alpha)$ is, as I understand it $E(\alpha)$ (are these the initials of odd & even?) and law I is therefore that the relation E if it exists between all the "elements" of a collection, exists between any part of them. Therefore $\star E(\alpha)$ asserts something concerning all the "elements" of α which is true of any two of them, if not of any one. It therefore asserts some agreement among them all. It asserts an agreement in some

(pages)

pliably suppose S to consist of an element, then no matter what its determination, it will agree with itself. But now saw I it tells us that by formal necessity this determination must agree either with the O of b , or with the determination of b_1 . This is as much as ~~to assert that there are only~~
two possible determinations in each respect, which we may call O and I . It also implies that the list of "essential respects" is the same for all collections.

So then I should describe your system as consisting of a free collection of "elements", a fixed collection of "essential respects" (let me call them "respects" for short) and of arbitrarily formed "collections" of those elements". Each element in each respect is determined in one of two ways. Let us arbitrarily take one element of each "collection" as first, and call its determination in each respect O , the other determination I . Then

§ 21. Let the determination of x in every respect be O . Then since $O(xy)$, the determination of y will be O . Then since $O(\alpha x)$ and $O(\alpha y)$ one element of α will be O and another I . Then in any respect the elements of α will be unlike or $O(\alpha)$.

§ 22 Since $O(\pi)$, in every respect some two elements of π will differ. Suppose then that in a certain respect $p_0 = O$ $p_1 = I$. Then the diverse collection will contain $r_0 = I$ $r_1 = O$ and thus in every aspect elements of π will differ $O(\pi)$.

§ 23 Let the determination of x in any aspect be O . Then by $O(xy)$, the det. of y will be I in any aspect. Since $O(\beta x)$, the det. of some element of β will be I , and since $O(\alpha x)$, the det. of some element of α will be I . Since α and β are diverse, the det. of some element of η will be O . Since in any respect the det. of some element of β is thus shown to be I , while the det. of some element of η is O , we have $O(\beta\eta)$. This is O word. Proof in Memoir 84 words.

§24. Since α consists solely of elements complementary to β , every element of α is in every respect different from some element of β . Since $O(\beta)$, in no respect is every element of α like every element of β . Since δ and γ are obverses, in no respect is every element of α different from every element of γ . Therefore in no respect is every element of α like every element of γ , or $O(\gamma)$.

There is one feature of your work which puzzles me; and it seems to me that this is because the explanations of the introduction are not sufficient.

The elements are "simple and homogeneous." Then I do not see how two can be equivalent, or how $O(p, q)$. I should think implies that two elements make up the system.

In §7 I learn that a collection "is determined wholly by the fact that certain elements do, while certain elements do not, belong to it." I should think, then, that $(x, r) = (x, x, r)$. Is that so?

Do your elements definite individuals, or indefinite individuals, or are they general? This is a vital question, the logic of the three being different. If they are definite, I do not see how one can enter a collection twice over. If they are indefinite, there are signs, and equivocal signs. May the same letter denote two different elements?

I should like a clear explanation of these relations and an accurate statement of the sense in which an element can enter repeatedly into a collection.

Is it that, besides elements and respects, there are also modes of composition, so that (a, b) and (a, a, b) and (a, b, b) and (a, a, b, b) etc are different compounds, though of the same composition? This would seem to be explicitly negatived

by 87.

very faithfully

P. S. Pearce

Of course the different equivalent elements differ in inessential respects.

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respect among a class of respects, which, to give it a name, I will call the "essential respects." Consequently $O(\alpha)$ the denial of $E(\alpha)$ asserts that in no "essential respect" do all the elements of α agree.

Law II asserts that if $O(\beta)$, that is if the elements of β do not agree in ~~any~~ essential respects, but $E(\delta)$ that is, if the elements of δ do agree in ^{some} ~~all~~ "essential respects", then there is some element b_n of β such that it ~~does not~~ agrees with all the elements of δ in some essential respect, by necessity. Suppose, for instance, that β consists of two elements b_0, b_1 . There is supposed to be no "essential respect" in which these agree. We may call the determination of b_0 in each essential respect O_i . Then the determination of b_1 , maybe in one "essential respect" 1, in another 2, in another 3, etc. (where 1, 2, 3 are merely distinctive marks.) Now in some essential respect the elements of δ ~~do~~ agree. For since

miting the determinations of any one element in a vertical column, those of any one respect in a horizontal line, each collection will be described by a block of numerals in the secundal system of arithmetical notation. The first column will be all zeros & may be omitted.

I do not see what use there can be in distinguishing respects in which no two elements are distinguished; and Laws I and II do not limit the possible combinations of determination. Therefore, as these two laws go, we seem to have just one possible respect for each number and one secundal place for each element of the system after the first. Therefore for 2^{n+1} possible elements there will be $2^n - 1$ respects.

Principles III, IV, V, VI come to this:
 For each element of the system, (and an element there is)
 three others each of which agrees with the
 first in one of three respects and differs
 from it in the other two.

1905 Aug 19

My dear Professor Joyce:

Receive your important Memoir today
& although I have not yet had time to read
far into Chapter II, I will venture on a few re-
marks which may for ought I know be con-
tained in the Memoir itself.

Beginning with the formal definition of an α -collection
in § 19, I will consider law I in the form

$$E(xy) \sim E(x),$$

that is, E expresses such a relationship be-
tween the "elements" of the ^{any} "collection" as remains
when some of these elements have been suppressed.
Further, since this relationship is one of equivalence,
the assertion $E(x)$ is that each "element" of α
after the first pair is suited to preserve the E -
relationship. It therefore expresses (does E)
an agreement among all the "elements". If I
ask what sort of agreement, the answer must be
an agreement in anyone of certain respects,
which I may for convenience refer to as essential

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respects. For if $a_1, a_2, a_3, a_4, \dots, a_n$ all agree in any one respect of a given class, so necessarily do any of these a 's fewer than n . Consequently $\Omega(\alpha)$ expresses that ~~they~~^{in any} "essential respect," the elements a_1, a_2, a_3, \dots , etc. are not all alike. Now let me see what limitation Law II places upon this interpretation.

That law is that if $E(\delta)$ then either $E(\beta)$ or else, ~~whatever~~^{for some} element of β b_n may have $E(\delta b_n)$. That is, if $d_1, d_2, d_3, \dots, d_n$ (the "elements") of δ all agree in an "essential respect," and if $b_1, b_2, b_3, \dots, b_m$ do not all agree in anyone "essential respect," then there must be some ~~one~~ b_n of the b 's, say b_n such that it does not agree with $d_1, d_2,$

d_3, \dots, d_n . This will necessarily be the case if the "essential respects" remain the same for all "collections." For, this being the case, the b 's do not agree with one another in that essential respect in which the d 's agree. Suppose, for instance we call the character of d_i in that

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respect α , so that $d_1 \sim \alpha, d_2 \sim \alpha, d_3 \sim \alpha, \dots$ then some b is α and some is $\bar{\alpha}$, or as I may write it, is 1. Then the former agrees in this respect with all the d 's. Thus Law II adds nothing to Law I except that the essential respects are the same universe of respects for all "collections," and there is merely a dual variation in each respect.

Let us form a table, or block, for each collection, ~~and~~ assigning a separate horizontal line ~~to each~~ "essential respect," and a separate column ~~to each~~ "element." β will denote the determination of the first element (the one arbitrarily taken as first) in each respect by α , and the same character α shall denote agreement with that first in that respect, while β shall denote the opposite determination. Then identifying respects in which no two elements differ, the possible "collections" of the system will be represented by blocks

HUG 1755.14

of numbers expressed in the secundal notation

00	000	0000
01	001	0001
	010	0010
	011	0011
		0100
		0101
		0110
		0111

Thus $O(\alpha)$ means that the block of numbers representing α does not contain zero. α will then might be expressed: "Elements which disagree with the same element agree with each other, in any one essential respect."

Of the special principles, III, IV, V express that, There are at least 3 different (i.e. non equivalent) elements in the system.

In each respect counted as essential there are elements that differ. (So that the whole system is an O -collection)

VI says: If there be an element w which differs from every essential respect from some element of a collection D , then there is an element of the system v which agrees with w in every respect in which all elements of D agree with each other and differ from w in all respects in which elements of D differ from one another.