

Sketch of the Infinitesimal Calculus

I. Introductory

1. Interest in the data of consciousness is the sole agency in the formation of thoughts. Without this interest, no reflection, no self-consciousness, no idea of external reality, no science.

2. Interest in consciousness results first in analysis of data, and in the distinction of a central or known point of consciousness from a vague and boundless (or better boundaryless) but finite surrounding field of consciousness. [2]

3. Interest in consciousness results, secondly, in the attributing of significance to this central or known datum of consciousness. To attribute significance is to regard this datum with more or less definiteness as the representative of a reality beyond itself; i.e., as the representative of past, future, or possible consciousness.

3. The past, future and possible can be known or conceived only in and through the present, and all there is in our idea of them beyond what is immediately and momentarily given, consists wholly in the indescribable act of projection, by which we regard the present as significant, in one or another of the mentioned fashions, of a reality beyond itself. The present is thus pro- [3] jected into a not-present.

4. The point of attention and of interest in consciousness is to inform an Unit. But for the consciousness of the moment in which this unit is formed, the point of attention is not known as an Unit, but only as a distinct content. The consciousness of the form as distinct from the matter is possible only in reflection. Only by Reflection, therefore, do we get the notion of an Unit.

5. In Reflection the present consciousness is referred to a past moment, whose content is conceived as ideally present in the present moment of reflection. The analysis of the data of consciousness is then an analysis of the form in which the conceived data are affirmed to have existed in the previous consciousness [4] that is now the subject of reflection.

6. The idea of an unit as stated an idea of reflection. The form of the act of distinction or of attention is reflected upon, and the result of this act is seen to be an Unit.

7. By *Unit* is meant therefore any content of consciousness that through a momentary act of attention, i.e. of interest, is brought into the focus or point of attention of consciousness; or again, any content is conceived as a possible subject of such an act of attention.

8. This definition, though it implicitly uses the notion of unit to define this same notion is nevertheless not truly circular. For any act of attention, though one, is not primarily known as one, nor upon the first reflection is the act known as one. The pro- [5] cess is this: Attending, we distinguish some content of consciousness. Reflecting, we perceive the content thus distinguished always to be what is meant by an Unit. Reflecting again we see that the act of constructing an unit is itself one. But not as being itself already

for consciousness an unit does any act of attention produce units. On the contrary, because in attending to our consciousness we produce units to which we clearly attend, therefore is the act of attention known to be itself an unit.

9. That anything to which we attend is itself a complex, and so may be further divided, that therefore we can attend to several things at once, does not alter the [6] above stated facts. For though I attend to three or a dozen impressions at once, yes, even though I could attend to a million things at once, still in that I attend to them all I have on conscious point distinguished from the vague surroundings of the conscious field, one object, however complex, and so the foundation of the notion of unity. No other idea of unity can be given but this, as a unit is what is distinguished in any moment of consciousness, from all else.

10. An unit may be regarded, in the moment when it is attended to, either solely as an unit, or else as also complex, potentially divisible into parts. In the second case, being now conscious of an unit, we regard it as possibly a subject of many distinct acts of attention, and [7] so view the one as also many.

11. An unite regarded as a complex of parts may be viewed as a definite or as an indefinite complex of parts.

12. If consciousness views the unit as, with respect to the past, the result of a synthesis of parts, consciously made by joining one part after another to form the whole, consciousness then regards the unit as a definite discrete complex, i.e., as a Number.

13. But not all units of attention can be so regarded. In many cases we are conscious that the unit present in consciousness is a complex, whose parts can conceivably form objects of [8] separate acts of attention, and yet we are not conscious of this unit as the result of any previous process of synthesis of part with part. Such is the case with units of time, of space and of motion. Such units may have very various material contents, but they all have in common these characteristics: —

1. They are conceived as not simple, but complex.
2. Their parts are conceived as possible subjects or units of other acts of attention.
3. The number of acts of attention that could exhaust the separate units is not conceived, and effort to conceive it fails.
4. The units are given to us as wholes, and could not have been constructed out of ultimate, i.e. not conceivably analyzable parts. They are not the results, but the starting- points of processes. [8]

14. Such units are Continuous Quantities. As starting points of processes they may be used to form discrete quantities. But they themselves are not reducible to discrete quantities.

15. How the notions of discrete quantity are built up does not belong to the present discourse except in so far as the process has thus far been suggested. But the infinitesimal calculus is concerned with the problems concerning continuous quantity. [9]

II.

The Measurement of Continuous Quantity

16. Thought, or consciousness to which attentive interest is joined, is primarily concerned with units. Passing from one act of attention to others it conceives itself as joining units together by voluntary synthesis into more complex units, or on the other hand as analyzing units into their component units. Thought is thus naturally concerned with discrete quantities, and in reflecting upon its own processes, [10] thought finds that all definite mental operations with quantity consist in the construction or in the analysis of discrete quantities, and that with continuous quantity as such no definite operations are possible. Continuous quantity is therefore always dealt with by reducing it in some way to discrete quantity. And as continuous quantity can only negatively be known. That a line or a surface or a motion is a continuous quantity known only by observing that my discrete analysis of my conception of it is never exhaustive, but that complex units are [11] recognized by me as remaining after all my analysis. A complete exact conception of a continuous quantity as such is never formed.

17. We can indeed in thought construct continuous quantities, but not out of ultimate units. The minimum result of our constructive process is itself a complex of yet undistinguished parts.

18. The first effort to reduce continuous to discrete quantity takes the form of measurement.

19. By measurement is meant the analysis of a continuous unit into a discrete number of component equal units assumed for the moment to be ultimate. [manuscript ends]