

Sketch of the Infinitesimal Calculus.

I. Introductory. ~~Notes~~

1. Interest in the data of consciousness is the sole agency in the formation of thoughts. Without this interest, no reflection, no self-consciousness, no idea of external reality, no science.

2. Interest in consciousness results first in analysis of data, and in the distinction of a central or known point of consciousness from a vague and boundless (or better boundless) but finite surrounding field of consciousness.

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3. Interest in consciousness results, secondly, in the attributing of significance to this central or known datum ~~of~~ consciousness. To attribute significance is to regard this datum with more or less definiteness as the representative of a reality beyond itself; i.e., as the representative of past, future, or possible consciousness.

3. The past future and possible can be known ~~and~~ ^{or} conceived only in and through the present, and all there is in our idea of them beyond what is immediately and momentarily given, consists wholly in the indescribable act of projection, by which we regard the present as significant, in one or another of the mentioned fashions, as a reality beyond itself. The present is thus pro-

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jected into a not-present.

4. The point of attention and of interest in consciousness is ^{inform} an Unit. But for the consciousness ~~that~~ of the moment in which this unit is formed, the point of attention is not known as an Unit, but only as a distinct content. The consciousness of the form as distinct from the matter is possible only in reflection. Only by Reflection, ^{therefore}, do we get the notion of an Unit.

5. In Reflection the present consciousness is referred to a past ^{moment}, whose content is conceived as ideally present in this ^{present} moment of reflection. The analysis of the data of consciousness is then an analysis of the form in which the conceived data are affirmed to have existed in the previous consciousness

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that is now the subject of reflection.

6. The idea of an unit is ~~defined~~ ^{as stated} an idea of reflection. The form of the act of distinction or of attention is reflected upon, and the result of this act is seen to be an Unit.
7. By Unit is meant therefore any content of consciousness that through a momentary act of attention, ~~is~~ ^{is} of interest, is brought into the focus or point of attention of consciousness; or, again, any content that is conceived as a possible subject of such an act of attention.
8. This definition, though it implicitly uses the notion of unit to define this same notion is nevertheless not truly circular. For any act of attention, though one, is not primarily known as one, nor upon the first reflection is the act known as one. The pro-

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cess is this: Attending, we distinguish some content of consciousness. Reflecting, we perceive the content thus distinguished ^{always} to be what is meant by an Unit. Reflecting again we see that the act of constructing an unit is itself one. But not as being itself already for consciousness an unit does any act of attention produce units. On the contrary, because in attending to our consciousness we produce units to which we clearly attend, therefore is the act of attention known to be itself an unit.

9. That anything to which we attend is itself a complex, and ^{so} may be further divided, that therefore we can attend to ~~several~~ ^{several} things at once, does not alter the

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above-stated facts. For though I attend to three or a dozen impressions at once, yes, even though I could attend to a million things at once, still in that I attend to them all ~~and~~ I have one conscious point distinguished from the vague surroundings of the conscious field, one object, however complex, and so the foundation of the notion of unity. No other idea of unity can be given but this, a unit is what is distinguished, in any moment of consciousness, from all else.

10. An unity may be regarded, in the moment when it is attended to, ^{either} solely as an unit, or else as also complex, potentially divisible into parts. In the second case, being now conscious of an unit, we regard it as possibly a subject of many distinct acts of attention, and

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- so view the ~~same~~ one as also many
11. An unit regarded as a complex of parts may be viewed as a definite or as an indefinite complex of parts.
 12. ~~Q~~ If consciousness views the unit as, with respect to the past, the result of a ~~series~~ synthesis of parts, consciously made by joining one part after another ^{to form} the whole, consciousness then regards the unit as a definite discrete complex, i. e., as a Number.
 13. But not all units of attention can be so regarded. In many cases we are conscious that the unit present in consciousness is a complex, whose parts can ~~not~~ conceivably form ~~objects~~ of

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separate acts of attention, and yet we are not conscious of this unit as the result of any previous process of synthesis of part with part. Such is the case with units of time, of space ^{and} of motion. Such units may have ~~various~~ ^{very various} ~~contents~~ ^{material} contents, but they all have in common these characteristics:-

1. They are conceived as not simple, but complex.
2. Their parts are conceived as possible subjects or units of other acts of attention.
3. The number of acts of attention that could exhaust the separate unit is not conceived, ^{and effort to conceive it fails.}
4. These units are given to us as wholes and could not have been constructed out of ultimate, i.e. not conceivably analysed parts. They are not the results, but the starting-points of processes.

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14. Such units are **Continuous Quantities**. As starting points of processes they may be used to form discrete quantities. But they themselves are not reducible to discrete ^{entities} ~~quantities~~.

13. How the notions of discrete quantity are built up does not belong to the present discourse, except in so far as the process has thus far been suggested. But the infinitesimal calculus is concerned throughout with the problems concerning continuous quantity.

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II.

The Measurement of Continuous
Quantity.

16. Thought, ^{or} consciousness to which attentive interest is joined, is primarily concerned with units. Passing from one act of attention to others it conceives itself as joining units together by voluntary synthesis into more complex units, or on the other hand as analyzing units into their component units. Thought is thus naturally concerned with discrete quantities, and in reflecting upon its own processes,

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thought finds that all ~~definite~~ ~~operations~~ definite mental operations with quantity consist in the construction or in the analysis of discrete quantities, and that with continuous quantity, as such, no definite operations are possible. Continuous quantity is therefore always dealt with by reducing it in some way to discrete quantity. And as continuous ~~quantity~~ quantity can only negatively be known. That a line or a surface or a motion is ^{quantity} continuous, I know only by observing that my discrete analysis of my conception of it is never exhaustive, but that complex units are

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recognized by me as ~~expressing~~
after all my analysis. A complete
^{exact} conception of a continuous quantity
as such is never formed.

17. We can indeed in thought con-
struct continuous quantities, but not
out of ultimate units. The minimum
result of our constructive process
is itself a complex of yet undistin-
guished parts.

18. The first effort to reduce continuous
to discrete quantity takes the form of
measurement.

19. By measurement is meant the
analysis of a continuous unit into ^(a discrete number of) com-
ponent-equal units assumed ^{in the moment to be} ultimate.