Introduction to
Sketch of the Infinitesimal Calculus
c. 1880

Historical Context

The manuscript is an incomplete eleven-page handwritten document.[1] The document is the second of ten in Harvard Archives Royce Papers (HARP) Box 60, all of varying topics from 1879 to 1881. The topic of the manuscript is a nascent phenomenology of consciousness to derive certain conceptions fundamental to mathematics, especially the concept of a unit and of continuous quantity. From 1878 to 1882, while employed at the University of California, Royce sought to systematically elaborate the theory of knowledge contained in his doctoral dissertation, *Of the Interdependence of the Principles of Human Knowledge*.[2] The project led Royce to a detailed investigation into the study of consciousness, and consequently, an increasingly pragmatic epistemology and nascent phenomenology.[3] A letter to William James dated 19 September 1880 summarizes some of Royce's conclusions drawn from his phenomenological investigations at this time. Royce, in a noteworthy development from the doctoral dissertation, dismisses the *Ding an sich* as unintelligible. Consequently, consciousness is the logical condition of reality: Everything real is relative to consciousness, and there is no reality outside of conscious experience. Hence, the critical method must begin with an analysis of the structure of experience from the standpoint of consciousness to determine how knowledge of reality is possible. Given the structure of experience is a lawful series of conscious states, whether possible or actual, then the problem of the possibility of knowledge is reducible to the problem of how a series of conscious states is knowable as a series. From the standpoint of consciousness, we are always conscious of the present and the present only. If the series of conscious states is knowable as a series, then every moment must be relatable to every other moment. Given such relations transcend the present, the living activity of the interpreter must willfully postulate past and future to ideally construct an indefinitely extending series of time within which the present is relatable to every other moment. The present moment is, in other words, a *unit* relatable to past, future, and possible experience through an ideal series of time postulated by the practical and volitional

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[1] All information on the manuscript is found in Oppenheim's Comprehensive Index, entry 82 in Part II. An online version is available at http://royce-edition.iupui.edu/docu/index.pdf.

[2] For an overview of this time period in Royce's life, see Dillabough, Joseph. 'Introduction to the Possibility of Experience' and 'Introduction to the Interpretation of Consciousness' in *The Writings of Josiah Royce: A Critical Edition*. February 2019. Available online at http://royce-edition.iupui.edu/transcribed-manuscripts/. An online version of Royce's 1878 dissertation is available at http://royce-edition.iupui.edu/online-royce-volumes/.

[3] For a sample of Royce's study of consciousness, his increasingly pragmatic epistemology, and nascent phenomenology, see c. 1880: 'The Possibility of Experience' and 'Interpretation of Consciousness,' available online at http://royce-edition.iupui.edu/transcribed-manuscripts/. See also a letter to William James dated 19 September 1880, in Royce, Josiah. *The Letters of Josiah Royce*, vol. 1, ed. John Clendenning. Chicago & London, University of Chicago Press 1991: 86-91. And the 1881 publication, 'Kant's Relation to Modern Philosophic Progress' in *The Journal of Speculative Philosophy*, vol. 15, no. 4. Penn State University Press: 360-381, available online at http://royce-edition.iupui.edu/online-royce-articles/.

aspects of the constitutive function of judgments. Within this context, the present manuscript is an early attempt at such a phenomenology of consciousness but with a mathematically-informed orientation as to how the present appears as a unit and how these units form continuous quantities.

Sketch of the Infinitesimal Calculus

The manuscript divides, in a geometrical style reminiscent of Spinoza, into two sections with a total of nineteen numbered propositions. The first section is an introduction with fifteen numbered propositions, while the remaining four numbered propositions are in the second section on the measurement of continuous quantity. The first three propositions are representative of Royce's pragmatic approach to phenomenological inquiry: We become conscious of those parts of experience that are of interest to our attention; and, without such attentive interest, we would never become conscious of anything. Our attentive interest reduces the indeterminate field of consciousness to a determinately known reference point — or *unit* — that acquires significance as a representation of past, future, or possible experience. Given these relations transcend the present, and we are only ever conscious of the present moment, then past, future, and possible experience are only knowable in and through the present. We are not directly aware that the present moment informs the construction of a unit. Reflectively, we distinguish between the form and content of the present moment as a unit and relate that unit's content with the content of some past, future, or possible experience. But, this relation is ideal: We conceive the content of a past experience as concurring with the content of some present unit; or, conceive of the content of some present unit as similarly concurring with the content of a future or possible experience. As representations of content not presently known, units therefore become *signs* of past, future, or possible experience. Thus, out of and within the present, the interpreter reflectively projects past, future, and possible experience by ideal relations of signification among units and their content.

A unit is a consciousness of content through attentive interest: Attentive interest selects some content present in consciousness and in reflection that content is recognizable as a unit with a form as well as a content. Of course, whatever we are conscious of is more or less complex. Yet, according to our attentive interest, these complexes are still units because anything to which we attend is a determinately known reference point that contrasts with the indeterminate field of consciousness. A unit is therefore conceivably one and many: As an element of attention or a complex divisible into parts by successive acts of attentive interest, but nonetheless a unit in either case. If successive acts of attentive interest synthesize parts with respect to past experience into a linearly ordered whole, the resultant unit is a discrete complex of enumerable parts, which is how we derive the concept of number. But, there are other units of attention. These are not synthesizable in successive acts of attention, nor constructible in reference to past experience, because no series of attentive acts could exhaust the number of parts necessary for their construction.

Consequently, these units are not simple but complex and always already given to us as wholes, since they are neither constructible nor conceivable as complexes of parts; therefore, these units are not results of but starting-points for the thinking-process. Such units are continuous quantities; for example, space, time and motion are continuous quantities. As starting-points for the thinking-process, continuous quantities, such as space and time, are used to construct discrete quantities, such as number, but are not ultimately reducible to discrete parts.

The process of thinking is the consciousness of units through attentive interest. The thinking-process either synthesizes units into complexes or analyzes complexes into constituent units. Such syntheses and analyses concern units of discrete quantities because these complexes have parts our attentive interest can exhaustively enumerate with successive mental operations. While units of continuous quantities are neither synthetic nor analytic because these complexes have parts our attentive interest cannot exhaustively enumerate, nor any number of mental operations can sufficiently comprehend. Consequently, we come to a knowledge of continuous quantities only negatively: We know there are continuous quantities because there are complexes that are neither constructible out of nor analyzable into discrete quantities. Therefore, a continuous quantity is a complex that does not have ultimate parts: For any part of a continuous quantity is itself a continuous quantity analyzable into yet further parts, and thus, no synthetic act could ever construct these parts into a single complex of continuous quantities. Despite the inconstructability and unanalyzability of continuous quantities, Royce believes an inexact conception of continuous quantities is constructible in thought through the process of measurement. Yet, the manuscript terminates before Royce addresses how measurement aids in the construction of such an inexact conception.

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