LECTURE III.

THE ACCESSIBILITY OF ABSOLUTE TRUTH.

I can not better begin this discussion of the accessibility of absolute truth than by citing a favorite instance of mine,-an instance found where you might least expect it, namely in a famous passage in one of the essays of Professor William James. It is a remarkable example of the comparatively unreflective decisiveness of James's own character that he should have furnished this instance of an accessible absolute truth at the very heart of a discussion of human fallibility and ignorance. I said in my first lecture, that I myself owe much to the activism of James's earlier essays. That the absolutism which I have been maintaining is but the sense of his own early doctrine, stated a little more fully than he himself stated it, I firmly believe. But let me give the instance, and leave it to your judgment to decide who is here the absolutist.

You remember in the I. volume entitled "The Will to Believe," James's several times repeated example of the wanderer in the mountain path. The wanderer is supposed to have made his way to a point where retreat prooves to be, so far as he can see, impossible. Of course human knowledge, that retreat is impossible is under such circumstances relative enough, but we are supposed to

learn that the wanderer so thinks, and that his further course must be decided in the light of this apparent ~~proof~~ truth. Ahead of the wanderer is a chasm which, so far as he is able to judge, it is just possible that he may leap if he tries resolutely. If the leap fails he will be dashed to pieces. If he stays where he is he will, so far as he can foresee, be doomed to starve or to freeze or to fall from his insecure position in exhaustion. The very essence of his situation is as you see, that all his knowledge seems thus far only relative and imperfect. The instance is used to illustrate a case where scientific methods of research cannot be applied by the wanderer himself, and wherein he seems as far away from a knowledge of absolute truth, as man in this dark world often is. James hereupon supposes the question to arise whether the man has any right to believe, in advance of scientific evidence, that he is able to leap the chasm. A successful leap seems the only probable way out. Now belief is a state of mind so associated with the tendency to be strong, so linked with the psycho-physical mechanism, that any person of common sense would say: if he believes that he can leap, he has a better chance to leap successfully than if he does not believe. Hereupon James also supposes that the man in question is in the possession of a certain resoluteness,-that he has some active control over his believing attitude, so that if he wills to believe that he can leap the chasm, he is likely to influence his belief, and consequently to have a better chance to escape. As a fact, if he were so resolute as not even to raise the question whether he can make a successful

leap or not, he might at once make his way to safety.

But now he is supposed actually to hesitate, and to raise the question: have I a right to believe in the absence of evidence, such as can be called scientific, that I can leap this chasm? The conscientious scruples that Professor Clifford expressed in a famous essay are supposed to come to mind. Does my duty to the truth permit me to treat as if it were a legitimate object of belief an assertion that is not warrented by the evidence now in hand? So the man is supposed to ask himself. James vigorously replies to this scruple that here is a case where the man's own will is a factor which may proove to be the decisive factor in determining whether or no the doubtful proposition is to be true. He suggests therefore that the man in question has every practical right to use the will to believe in his own power to help him over the chasm if he can. And the moral of the story of course depends in part upon this comment.

Up to this point every bit of knowledge and of comment that is supposed to be present or to be introduced into the argument has to do with wholly relative matters,- matters of our imperfect knowledge. When you are in doubt and cannot decide upon the basis of any absolute knowledge, you have still a right to believe when such belief itself may proove a factor in making true the proposition that is in question. So much about James's position is clear to any reader. But suppose that all these considerations do not serve to overcome our wanderer's too highly trained scientific conscience.

Suppose that he says: "Since I have no right to treat as true a proposition that I do not know to be true, and since I have not the evidence to decide this issue, I must not decide that I ought to will to believe in my power to jump the chasm. I must leave the matter undecided. I will not will to believe. I will not introduce the factor of my belief into the situation. I will decide nothing. I will leave the facts to decide the issue experimentally, as facts decide issues in laboratories. Whenever it becomesclear to me that I can leap I will leap. Till then I decide nothing."

But here at last James intervenes with what is in effect the mention of an absolute truth. He points out that "Not to decide is itself a decision. Everything else may be doubtful, all else in this dark world may be unknown to the wanderer. But this he can see: If he decides to remain where he is, if he decides not to decide in favor of the will to believe, then the moments are flying, probable starvation and freezing creep nearer, the last chance for safety may be lost while he waits with ebbing powers. To decide to wait is, of course, not to decide with certainty what the outcome will be. Our knowledge of the world is limited. Angels or human rescuers or involuntary impulses may intervene to save the hesitant. The chasm may be closed by magic. About such things we may be assumed to have only relative knowledge. But each momentary decision is irrevocable. And at any moment, so we repeat, Not to decide is a decision. The will not to believe is itself a will. And the will not to believe may proove the one fatal factor that ends

the wanderer's quest. I have restated the case in my own words.

Not to decide is a decision. Hames's observation supposes a man who is deliberating, and whose will can intervene to alter the situation. Granting this supposition, the comment expressed in the foregoing words embodies an absolute truth. In other passages in the early essays, both in the essay here under discussion, and elsewhere where he is considering the problem of free-will and certain problems about faith and life and the universe, James substantially uses the same essential consideration. When we have not sufficient knowledge to decide certain issues, we have still the responsibility of a decision resting upon us, whenever the situation requires of us an act, and when ever we ourselves are in any sense in control of this act. For if we say: "Ignorant as I am I cannot act," that decision is itself an act. And this proposition holds true whatever may be your theory about the nature of the will, and whether you believe in free-will or in predestination, or in any form of psychological determinism. For the question is here not one of causation but one of intent. If I in any sense do what I intend to do, then the resolution not to have an intention is itself an expression of an intention. This proposition is absolutely true, simply as a reflection upon the very nature of the decisive will, and of the situation in which the decision is called for. The proposition, viewed apart from its context, might appear a commonplace, or a quibble. James's early

essays, full of unconscious absolute truths regarding the nature of the ~~ÆÆÆÆ~~ active life, regarding the significance of decisiveness, show how vastly significant the implications of this absolutely true proposition are. ~~¶~~ Not to decide is a decision. That is, you cannot be a neutral, when life and your own will~~Æ~~ cooperate to present to you alternatives between which you must choose. The situations in which this truth applies are vastly more numerous than those which James expressly insisted upon in his essay, although in spirit he has very richly developed the consequences to which his own attention was directed. Put yourself back in imagination to the beginning of the civil war. Conceive yourself already committed by obligations already assumed to the military service of your country in case it should be endangered Conceive yourself facing the issue that so many West Point men faced: Is the federal government, or is my own state government the representative of my country? Who is right in this controversy? Many a man might say: "I do not certainly know how to answer these questions. The subject is too vast for me, I am ignorant. I will not decide, for I have not the evidence. Let me then evade service by going to Europe until this unpleasantness blows over." At this point there would once more appear what we may call James's principle. We should have to say to the doubter once more: You cannot evade the decision. To go to Europe is not an evasion, it is a decision, which certainly leaves you false to any obligations whatever that as a military man you have already assumed. There may be doubt whether a decision for the North or for

the South would be a traitorous decision. But reflection will show that the one sure way to commit treason, in case you are indeed bound to the military service of your country, is to go to Europe.

For precisely similar reasons the main issues of the moral life, wherever they may arise, and our decisions with regard to our standing towards our social order, its great questions, its religion, and its similar interests, are subject to James's principle. Present me an issue involving a matter to which my will is already committed, an issue such as the saving or the conduct of my life, my bearing towards humanity, towards the universe, towards the working out of the principles of any science that I have once begun to investigate, for the reasons upon which James dwelt neutrality is impossible. Of course I may suspend my judgement so long as a particular decisive moment has not been reached. But when the decisive moment comes, not to decide is itself a decision. Of course such a decision, namely a decision not to decide a presented issue, a decision to declare that since I do not know so and so, I will act only under the conscious limitations imposed by this ignorance, such a decision may be the right one in a given case. So the physician in doubt about his patient may adopt a waiting policy and in so far suspend or altogether decline medical interferences that would be advisable if he knew more. But the absolute truth is that a waiting policy is also a policy, and that if one adopts it he adopts it for a

reason, which he must be able to defend upon known grounds. It is often a symptom of expertness to know when to suspend judgment. But deliberate suspension of a given judgment is itself expressive of the decisive will to suspend permanently or temporarily a given sort of judgment. I may decide to abandon a church fellowship because I do not know whether or no the creed of the church is true, but such a suspension of judgment expresses itself in the decisive deed of the nonconformist.

An able sea-captain once remarked to me, as he dropped anchor in a fog just inside the Delaware Breakwater: "It is a great part of my business to know when not to go on." This was an expert's expression of judgment. It resembled the caution of the physician or of the scientific enquirer in a region where the mists of ignorance are too thick for present decision regarding certain matters. Such an act expresses what one might call the will not to believe. And the will not to believe may in a given case may be the only rational will to observe. But still my captain's decision was fully covered by James's principle. The captain anchored. He thereby took upon himself all the responsibilities for delay, and for any possible mishap that might proove in the sequal to have resulted from the anchoring. The decision was unquestionably wise. It was due to ignorance, and to ignorance dealt with in a tentative way. But it was subject to James's absolute truth

II.

The foregoing type of cases will go very far to illustrate the sense in which as I should maintain absolute truth is in fact accessible to us. The nature of the foregoing discussions in the previous lectures will serve at once to explain why I choose to start with the case that some of you might regard as merely a discussion of maxims of practical prudence. The intimate connection between the practical and the theoretical truth which athe foregoing lecture at length discussed will show you how important in my opinion such modes of access to absolute truth will proove. If the foregoing account is correct, absolute truth is not something abstract or separate from life. It belongs to our judgments when they are considered in their relation to the decisive will, as well as in their relation to those objects in which a decisive will is interested. You want to do something, to that end you express yourself in voluntary deed, and you guide your deeds by your judgments. Your judgments characterize and interpret objects, and counsel your will to adjust itself accordingly. Your judgments are irrevocably true or false in so far as they give counsel that implies deeds which are either hits or misses. The distinction in question between true and false judgments is absolute, because of the irrevocable character, of your deeds, and because of the individual significance of your life in its place in the world wherein you intend to do your deeds. Now the enormous

complexity of the resulting truth relations makes the truth itself the object of an ideal conspectus, which you virtually conceive as estimating your judgments in relation to your acts by your survey of your whole life. Such a conspectus as we saw at the last time, is necessarily to be conceived as supratemperal. No human being under present conditions ever attains to this conspectus as he conceives it. Therefore absolute truth remains in the most of our lives an ideal,- or as Kant called it a regulative principle. Yet we are not without one mode of access, always an imperfect access, to the nature of this absolute truth. For after all we are dealing with our own ideals. And the nature of this ideal is determined by the nature of our own will and intent. In so far as we have some such test of the nature of our own ideal and intent as the one which James illustrates, we have a certain insight into the nature of absolute truth. The will intends to be decisive. Our judgments intend to characterize the life of this decisive will. When we come upon a principle regarding the nature of the decisive life which can be tested as James tests his principle, by noting that the very effort not to think in accordance with this principle and not to decide after a fashion which will express this decisive nature of the will, is an effort which logically as well as practically defeats itself, then we are certainly on the trail of absolute truth. We are on the way towards grasping the meaning of life, the essence of the will, that which the conspectus of our life will itself observe to be the reality of life, if ever our life is the object for such a conspectus.

So while I here lay no stress upon my own feeling of certainty. I do point out that in my opinion James's principle involves a real access to the nature of absolute truth. That which you do even in deciding not to do it is something that must be willed whenever the will is in question at all. And the life that is characterized by such a will has knowable features, and very interesting ones as James's essays so well bring out.

From the cases now considered I can return to a group of cases suggested by the discussions of our first lecture. The ideal of the decisive will, as we said, includes the assertion, that every deed of ours once done is irrevocable. A perfectly fair question arises as to whether this universal assertion is itself an absolute truth. When I uttered the assertion I told you that I did not regard it as immediately certain. I said that it was of use in defining the ideal of absolute truth, and as so much of the ideal of absolute truth is of value for the purposes of every decisive person, whatever his calling. But now suppose that you imagine the principle not to be true. Conceive that some deed of mine is revocable. Conceive that I do something and then undo it, so that it really is true that that deed having been undone, becomes so that the world of reality past as well as present simply has no place whatever for that deed. It requires comparatively little reflection to see that if I try to make such a conception concrete I am placed by my own act in a position of the following sort: I am to do something. For instance I am to sing a song

once through. Then I am to unsing that song. That is I am some how to make it true that that song never was sung. Let the feat be accomplished by whatever magic you please. Then by hypothesis it is true, or becomes true, that just that individual singing of just that individual song by just myself, never took place. But in stating the supposed truth, I seem to have contradicted the proposition uttered. For to what individual singing of what song by whom do I refer? Briefly I refer to what was sung. And I now say that what I declare to have been sung is also and equally declared not to have been sung. In willing to believe, or in hypothetically affirming this proposition, I have withdrawn my will as I willed it, I have taken back my utterance as I uttered it. I have not asserted anything. I have not meant anything. In brief the denial of the proposition that the deed once done is irrevocable is a denial that denies itself. In vain do you call such considerations abstract. They enter into the most concrete life of common sense all the time. The irrevocable character of a deed is one with its very nature as an individual deed. If you deny this irrevocable character you affirm it, even in pointing out the very individual deed whose irrevocability is to be denied. Once more I am laying no stress upon my feelings of certainty in this matter. I am asking you to consider it as fairly as you will. Whoever comes to me and says that he has found an undone deed, that is a deed

that having been done became undone, rouses the reasonable question when was that deed done, and who did it. If he answers by asserting that he refers to that individual deed done then by so and so, the deed by bypothesis is not undone but done. Here in brief is a proposition whose denial implies its reassertion. We seem to be dealing with absolute truth. I hear you exclaim indignantly : "Mere logic-chopping, mere abstraction, such dialectic can enlighten nobody." This logic-chopping I reply: The logic may be dreary. but how momentous the truth is, all life exemplifies. Look back on your own life and consider for an instant the tremendous import of the word irrevocable. All the horror and all the glory of our existence are bound up with it,-inseparably. This truth then is not barren. And it certainly is not the invention of mere intellectualism. And since it is a truth whose denial reinstates it, I conceive it to be very much like James's principle, an absolute truth. I leave its further contemplation at this point to your own judgment.

Closely connected with the irrevocable character of the deed is that absoluteness of the distinction between yes and no, wherever an issue is sharply drawn, and absoluteness upon which I have dwelt in both of the foregoing lectures. One of the first, if not the very first, really exact idea that a child can get, is the one that is expressed when he begins to say no. The conception in question is for a time at least very interesting for most children. The not-relation is, as I need not say, the most essential relation in a very customa-

ry view in a system of logical relationship. Of that system of logical relationships I will speak further in a moment. But as you are all aware we make use of the not-relation wherever we wish to classify, to define, to be precise, to join determinate issues.

Now you are well acquainted with the tradional so-called laws of thought: The principle of contradiction, the principle of excluded middle, which in combination with the law of identity are sometimes viewed as the most fundamental statements in all logic. I am quite ready to point out that modern exact logic does not make the same use of the principles of contradiction and of excluded middle as the older logic did in its presentation of logical principles. As a fact there is one way of viewing the matter, and to my mind a good way, which regards the principles in question just as a convenient mode of defining the not relation. That is, if you want to distinguish between A and not-A in an exact way, you may say that they are so related that nothing is both A and not-A, while everything is either A or not-A. This way of proceedure defines the not-relation in terms of the conceptions of <u>both-and</u> and <u>either-or</u>, as well as in terms of the conceptions of <u>nothing</u> and <u>everything</u>. As a fact the various elementary logical conceptions are mutually interdependent. All of them can be understood in terms of a properly selected group consisting of some portion of them. And the various selected groups of fundamental logical conceptions can be somewhat arbitrarily chosen. Yet however you define the not-relation, there can in the end no way of escaping the recognition of its fundamentally important character,

and of that sort of absoluteness which the principles of contradiction and of excluded middle attempt to express.
People who do not like logic often say that they do not like to live under the rule of some logician's principle of contradiction, and declare that the principle is too abstract to be the expression of an absolute truth. But as a fact the not-relation is a most practical one. Its nature is determined by the decisive will. The principle of contradiction is when expressed in terms of the will and instance of the same character of the decisive will that is otherwise expressed in James's principle. No difference could be more concrete, and less the creation of barren intellectualism, than the lifference between voluntarialy signing a note and not signing it, or voluntarily consenting to a purchase or refusing it, voluntarily bidding at an auction or not. The broker's boy in the market-place and surely the courts of law, find much use for the absoluteness of the distinction between yes and no, as do the formal logicians. In fact formal logic seems to me to be simply the theory of certain forms of the decisive will, and of the objects, such as classes and relations, which are defined in terms of the decisive will. In all such regions as I submit we have access to absolute truth precisely in so far as we have access to the nature of the will in terms of which the ideal of absolute truth is defined. But that upon which I must here insist is that such knowledge is not barren or merely formal.

It is a knowledge of that upon which all that is practically decisive in real life depends. If you want to find out the difference between yes and no, enter upon any enterprise which involves decisions. But surely in the light of the discussion of the last lecture I may be spared from further consideration of the question whether the truths about the distinction between yes and no, and the other fundamental logical conceptions, are truths which merely happen, or which are merely dynamic. They are ~~XXXX~~ truths which so inform the nature of real life that everything in heaven and earth, viewed temporally or viewed supra-temporally illustrates them and embodies them. They are ~~XXXX~~ neither merely static, nor yet merely dynamic. Nor are they true merely because they work. Yet without them is not anything made which is made.

III.

From the mention of the fundamental logical relations, we may turn to a brief consideration of the use which is made of these relations in the work both of the inductive and the deductive reasonings upon which the scientific methods are based. It is the repeated boast of the most prominent representatives of recent pragmatism that their own view is but an actual generalization of the methods and of the views regarding truth which have always been more or less consciously entertained by the investigators and students of the special sciences that deal with nature. The working

hypothesis, so they say, has long been known and used, and is daily used in the proceedure of every empirical science. Science is the attempt to learn how to control experience. It is successful in so far as it discovers hypotheses that actually work within the range of observation. It does not hope to discover absolute truth. It knows that such and such successes have taken place, that such and such predictions are verified, that such and such views about the nature and behavior of things meet empirical confirmation as far as we go. Science is content with the relative and with the empirical. The absolute not only escapes its jurisdiction but does not arouse its interest. The inductive methods are simply the methods of common sense sharpened defined, rendered more precise, and adapted to the various types of facts with which the sciences deal. And pragmatism which declares that propositions are true only in so far as they work is an extention of the scientific method into the philosohpical field.

In the previous lecture I pointed out how vague in many respects the phraseology concerning the working of hypotheses, such phraseology as currently used, appears to be. The exact testing of hypotheses such as is attempted with the aid of instruments of precision in the more developed natural sciences, is a testing which is very different from the mere following of such workings as might at first occur to mind, when one crudely grasps the general sense of the hypotheses. The scientific of hypotheses depends upon deducing from them the

consequences which must be true in case the hypotheses are true. Now a physical or a psychological working of a belief is by no means identical with an exact deduction from a defined hypothesis. And I do not think it unfair to say that one misses in the recent literature of pragmatism any close attention to the details of the process of deduction as that process appears in the more developed sciences. If more attention had been paid to the nature of deduction itself it would have appeared to the exponents of pragmatism that an hypothesis cannot be exactly tested unless exactly so much of absolute truth is accessible as is necessary to understand such implications of an hypothesis as are needed for a test. One too easily supposes in a general survey of a subject that to deduce the consequences of an hypothesis is essentially a simple, and not a very important thing, in case one is fairly equipped for the work of the special science with which one is dealing. I do not believe, to be sure, that such is the actual view of the trained workers in those science which have reached a high degree of theoretical development. Students of physics always express great respect for the deductive or the theoretical side of their own pursuit, and well know both how difficult and important it is. Students of the biological sciences, who are indeed well acquainted with the vast importance of a precise technique, and of instruments of sufficient precision for the recording of their observations lay great stress upon this technique itself and are aware that testing an hypothesis involves a great deal of thought-

fullness, and that this thoughtfullness has to be guided by rules which are as a fact highly logical, although the students of such sciences are frequently little interested in the technical logical formulation of these rules by means of which they think. To be sure as soon as statistical methods enter a branch of biological science, the mathematical, that is to say the logical, aspect of methodology comes in sight, and those concerned face problems which involve decidedly formal abstractions and extended deductions. But in general the workers in such sciences much as they prise their technique and their training as thinkers do not spend much time in thinking about thought. And hence they do not serve as very useful guides in helping the student of scientific methodology to understand how far the special sciences make use of the concept of absolute truth, and how far these sciences depend upon viewing some absolute truth as accessible.
It is on the whole comparatively easy for persons who have a dislike for exact thinking that the vague phrase; an hypothesis is true if it works, and only so long as it works, is a fairly adequate account of the methods actually in use in the various empirical sciences. But for our present purpose we must look a little more closely although indeed in our brief space very superficially at the way in which the actual work of the sciences of experience is related to the accessibility of absolute truth.

IV.

The best general theory of the process of induction is the one that has been outlined by Mr. Charles Peirce. The relation between deduction and induction in the sciences have been no where more clearly stated than by him. While as is well known the name pragmatism is due to Mr. Charles Peirce's initiative, it is well known that Mr. Peirce is no pragmatist in the later sense of that term, and is on definite record in opposition to its view of the nature of truth. It is well known to those who have examined Mr. Peirce's discussions of this subject that he has freed us from the current dogma of the text-books of inductive logic according to which induction depends upon the apriori assumption of the uniformity of nature. In so far Mr. Peirce is a thorough going empiricist in his logic of induction. On the other hand he has given us a precise definition of the concept of probability which may be recommended to all those who have not clearly distinguished between the predicates probable and true, when applied to a given proposition. There is no time here to expound Mr. Peirce's theory. A few points only are here of service for our momentary purpose. Inductive reasoning is a concept identical with the concept of probable reasoning. When a conclusion does not logically follow from certain premises, but is made probable by them, the conclusion is the result of a process of induction. The concept of probability depends for its exact definition upon the concept

of truth. It is infact, as Mr Venn has also insisted an essential statistical concept precisely as it has an exact and scientific character. Of course the popular and unscientific use of the term probability as a name for whatever more or less vaguely defined degree of belief one happened to possess regarding a proposed assertion, remains in actual usage. But so far as possible in a scientific investigation one endeavors to approximate to a statistical conception of probability. And of course this statistical conception is in the concrete familiar to all students of statistical branches of science. It is Peirce's especial merit to have defined the whole logic of induction in terms of this concept. To speak in general terms one can assert that a given proposition A has a probability p in case the proposition A belongs to a class, whereof the proportion p are in the long run true. Of course one must have reasons for classifying the proposition A with the class in question rather than with some other class; but in regard to the nature and the validity of these reasons I cannot here further speak. From the nature of the case the probability of a given propositdon which is not known to be true depends upon our classification of the proposition, and on our exact or inexact knowledge of the statistics of the class to which it belongs. And that is why the predicate probable as I said in my opening lecture is subject to such prodigious variation with the changes and with the growth of our knowledge. And nevertheless the predicate probable gets its exact meaning

so far as such exactness is possible at a given stage of our knowledge in a way that is decidedly independent of our private state of beliefs. One who studies probabilities by direct induction, without the use of hypotheses, does so by selecting as fairly as possible samples of some large class of facts in terms of which the probability of propositions is to be estimated and by substituting the sample in his reasoning for the whole class sampled. Thus if a certain percentage of men of a given age who appear to be in good health are found in some considerable sample of fairly chosen men to die within a given time, say a year or ten years, one defines a statistical probability and for purposes of insurance may assert, subject to correction, that the proposition: This individual man will die within a certain time, has a probability defined by the statistics of the chosen sample. The methods here in question are in countless instances familiar. They do not presuppose the concept of uniformity as a basis for the estimate of probability. It is Peirce's merit to have made this especially clear. The methods of reasoning which are thus indicated are the only ones applicable so long as the science is in the statistical stage. The probabilities definable only in these terms are subject to constant correction. They are indeed accepted as valuable practical guides, and in so far agree with the view of the inductive process which the pragmatists emphasize. But what the current pragmatism does not emphasize is that every elaboration of statistical probabilities of this type depends upon the accessibility of numerical proof. All the concepts in terms of which

statistical results are marshalled are themselves mathematical concepts. Without mathematical truth, no definable statistical probability. Experience alone furnishes the data which are to be sampled, statistically analyzed, and used to define the definite probability. But without mathematical, that is without logical deduction from numerical premises, the statistical data would remain beyond analysis, and the resulting probabilities would be wholly vague.

But if we pass from the more direct forms of inductive inference to the higher stages of induction, our method depends according to Peirce's analysis, upon the more or less expressly statistical sampling of the consequences of hypotheses. Suppose I have a definite hypothesis, from which calculable results can be deduced. Then if the hypothesis is true, these results ought to be found in experience. If no other hypothesis is known, or in the present state of knowledge is possessed of any high antecedent probability,-no other hypothesis that would give these results, then if these results are found in experience, the hypothesis not merely in the vague sense works, but appears as belonging to a class of hypotheses of which for general logical reasons we can say that the majority of them are approximately true, or possess a relation to the truth which can be probably and approximately estimated. Instances, well known to common sense, suggest the general character of the reasoning here in question. Eclipses are calculated and predicted in advance with very great accuracy.

One can say, of course if one pleases, that the hypothese usel for predicting the eclipses by the astronomers actually work, but a more exact although inecitably complicated account of the matter is ottainable upon the basis of Peirce's concept, if one defines in general statistical terms the probability that any hypothesis except the group of hypotheses used by the astronomers would serve to give not only the successful computations of the eclipses but all the other phenomena that the Newtonian theory of gravitation enables us to predict. The result, as pointed out in a previous lecture, is that while Newton's theory can never by any possibility be prooved to be true, we can show that the movements of the heavenly bodies are subject to laws which probably do not vary from the Newtonian law by more than certain very small amounts in the definition of certain physical quantities. The whole result remains indeed only probable and only approximate. But upon theoretical, that is upon logical grounds, it can be shown that the probability is very high and the approximation very close.

All this I say holds true on the basis of the view that one can accurately deduce from certain hypotheses the precise consequences that would follow from them if they are true. Precise and somewhat extended deductions thus become necessary as a preliminary and basis for the verification of hypotheses and for the establishment of the sense in which a given hypothesis may be said to work. Always on the higher level our

interest is not whether the hypothesis merely meets our vague expectation, or receives in a given state of our minds sufficient empirical illustration to sustain our private beliefs. On the contrary the issue is whether the deductions that are true if the hypothesis is true agree with new experiences, which are collected solely with the view to submit these deductions to the test.

V.

The result of this inevitably very summary sketch of the situation which the inductive sciences present to the logician is this: The only workings of an hypothesis which can serve as tests whereby a determinate and fairly objective probability can be given to hypotheses are tests that depend upon deduction. But deductive reasoning is reasoning which undertakes to show, not what is probable, but what is certainly true in case the given premises are true. Without a comparison between the results of such deduction and the data of experience no determinate probability is definable. In otherwords without such deduction one is left with mere vague satisfations and dissatisfactions upon one's hands. Apart from statistical analysis, and logical marshalling of the data, the workings of an hypothesis in the more direct instances of simple induction are left to the mere estimate of our feelings. And the results remain unscientific. In a vast number of cases however where the formally statistical methods are not used, an hypothesis acquires scientific value by virtue of the fact that very numerous consequences that can be deduced from it can be verified, and that these consequences

appear unlikely to follow with equal ease from other and con flicting hypotheses. But in all such cases the probabilities in question are definite only in case the deductions from the hypotheses are exact. Without precise deduction, no definitely estimable induction. This is a rule which can be illustrated in the most various ways in all those portions of science where one has got beyond the simple statistical enumeration of cases, and has reached the stage where general laws are definitely in question at all.

Our interest in all this lies in the fact that however limited to mere probabilities our knowledge of nature may be, deduction is concerned with getting access to absolute truth. For the assertion that a proposition A involves a proposition B is for reasons set forth in our foregoing lectures an assertion about a matter of absolute truth. One assertion implies another or it does not. A bit of deductive reasoning is either right or wrong. And the existence of the exact and deductive mathematical sciences is a proof that we have in a considerable measure a genuine access to absolute truth of the deductive type.

The nature of mathematical science itself has been until recently one of the most generally misunderstood of the topics in which philosophers are interested. Professor Benjamin Peirce father of this Charles Peirce first announced the often quoted assertion that mathematics is the science

which draws necessary conclusions,- in other words that mathematics is coextensive with the range of precise deductive reasoning. Some processes of deductive reasoning are so brief that we express them in a sentence or two and pass on to something else. In such cases we do not speak of mathematical science. But if the deductive reasoning is extensive, and particularly if the range of the subject matter is sufficiently complex to make the use of symbols convenient, then the reasoning becomes explicitly mathematical. Ther is no sort of necessary connection whereby mathematical science is limited to the study of quantitative relations. Any exact set of relationships, or in the sense of our two foregoing lectures any set of exact and coherent expressions, resolutions, and decisions of the will can form the basis for mathematical reasoning. In science the special use of mathematics is to work out the consequences of hypotheses. Mathematical science proper is not concerned with the truth of its initial hypotheses. It neither regards them as axioms, nor attempts to verify them empirically, beyond bringing them into such connections with the world of known truth as shall serve to render these hypothese interesting enough to investigate. Mathematical science proper is concerned simply with reasonings of the form; If A then B. But for reasons indicated in the foregoing discussions, precisely such assertions as: If A then B, are absolute truths if they are true at all. If mathematical science is possible, absolute truth is acceptible.