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Lecture II

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Lecture II. General Survey of the Concepts useful in Various Sciences.

In the former lecture we sketched the general problems to which this course is to be devoted. The starting point of our whole inquiry is furnished by the observation that widely sundered branches of scientific inquiry, although they may deal with what at first seem to be extremely various sorts of facts, still find, in many cases, the same types of concepts useful for the purpose of dealing with these different [2][1] regions of fact. In order to exemplify this wide range of usefulness which certain concepts possess, we briefly considered three typical instances of the principle thus enunciated. As we did so, we asked ourselves the questions: Why is each one of these concepts so widely and so variously applicable? Does the reason lie each time in the nature of external things, or in the nature of our thinking-process? We began to see that such questions are not easy to answer, and that the effort to reply to them takes us into very deep problems of philosophy.

On the present occasion, I propose to begin the task of surveying a little more systematically [3] the most important forms of concepts which human thought finds useful, either in all its investigations, or else, at the least, in very or numerous and in widely sundered types of investigations. At the last time, first, the numbers, and especially the whole numbers, next the more special concepts of the statistical sciences, and thirdly the concepts of rhythmic processes, were our chosen instances of what one might call general conceptual forms, such as are serviceable in sciences of the most various kinds, and such as help thought to master very widely contrasting types of facts. We now want to become more closely acquainted with several of the most general conceptual forms, which you meet with, [4] so to speak, anywhere, or almost anywhere, in the whole field of science. Of these most general conceptual forms you will soon see that the instances which we employed at the last time are but special cases.

[1] Numbering restarts in manuscript.

By the term conceptual form, I shall hereafter mean, as I may say, any concept of very wide application in the work of thought in various departments of research.

Let me begin my present sketch of the generally used conceptual forms by first bringing to your memory what you all know about the conceptual forms which our thought invariably uses, whatever may be the topic about which we are thinking. Then I may go on to mention certain less immediately obvious conceptual forms which science also uses in its various investigations. [5]

It is a commonplace of the text-books that whoever thinks, does, with the facts about which he thinks, two things: First, he classifies these facts. Secondly, he attempts to discover the relations of the facts. These two processes are very closely interconnected. We are in fact often tempted to view them as constituting but one process. And they do indeed constitute two aspects of a single process; but these are aspects which it is often convenient to distinguish. To classify facts is to arrange them in various ideal collections, which for certain purposes we keep apart from one [6] another in our minds, enough to assure us that when an object belong to one class it does not belong to certain other classes. These collections, or classes, may be small, as may, for instance, the groups of students who take some one advanced course in a University. But, on the other hand, the classes which the thinker considers may be indefinitely vast ideal collections. For instance, the class triangle is an ideal collection of geometrical objects which comprises an infinite number of possible instances. In general, the most frequently used class-concepts in the more exact sciences stand for very large ideal collections of objects. The same object may belong to many classes. But every class is opposed in our minds to certain other classes with which it has no object in common.

On the other hand, when we mention the relations of facts, we usually first think of taking facts in [7] very small groups, and then of studying what we call the various single relations amongst the facts that are found within each group. In a very frequent type of cases we deal with a group of two objects only, ie., with a pair of objects. We then note that one object of the pair has a certain determinate relation to the other; as, for instance, we observe that one man is taller than another, or that one light is brighter than another, or we assert that John is the brother of James, or that A is B's debtor. Here the noting of this single [8] relation, taller than, brighter than, brother of, debtor of, depends upon first considering the pair of objects between one of which and the other this relation exists. If we begin with a very large group of objects, such as a crowd of men, and asked "Who is the tallest man"? or "What ones amongst these men are taller than others?", we could not deal with the question considerately until we had first broken up the large group into small groups, and, in fact, into pairs, and had repeatedly asked the question: "Is this man taller than that man?"

When we talk of classes of facts, [9] our ideal groups thus tend to be large and may be infinitely, or at least indefinitely large. Moreover, to classify facts means for certain purposes, to set facts apart in different groups, to sunder one set of facts from another. When we talk of relations, we consider small groups of facts, and in many cases, we take account only of pairs. Furthermore we lay stress not upon the [illegible] connection or link of the facts. There then constitutes the contrast between the conceptual forms Class and Relation.

Yet this contrast between thinking in terms of classes of fact, and thinking in terms of the relations of facts, is no absolute contrast. Where we have to do with classes of objects, there we also have to do with relations. Whenever we consider relations, we are inevitably led to form classes. Thus, for instance, every member of a class is seen to stand in a peculiar relation to that class, so soon as the class is viewed as a sort of collective entity. This relation of the [10] member to the class is the relation of belonging to the class. Thus John belongs to the class man; Socrates belongs to the two classes, wise, and Athenian. And further, every member of a class stands in a peculiar relation to every other member of that class, the relation of being fellow-member of that class with the other member. Thus John and James are fellow-men; and this establishes a well known relation between them which the moralist and the religious teacher may find to be of inexhaustible significance. So classes are impossible without relations. [11]

On the other hand, when we think of relations as such, we are at once led to define classes. Thus, as soon as we learn to conceive that “John is the brother of James,” we are led to the definition of the class of beings called brothers. A brother is a man such that there exists some other human being, man or woman, to whom the man first mentioned stands in the relation of being the brother of the other. Thus the consideration of the single pair of human beings wherein the relationship called brotherhood gets illustrated, leads us to define the indefinitely large ideal collection of the [12] men who are called brothers.

Classification, then, and the definition of relations, are closely interconnected processes. Both occur in our minds whenever we think about objects. And so the conceptual forms Class and Relation, are forms of universal application in all our sciences. But for the moment, I will say no more about these two forms, viewed in their abstraction from other conceptual forms; because I do not want to weary you at the very outset with these apparently simple abstractions. We shall soon have reason, however, to return to these fundamental conceptual forms [13] with certain more concrete problems in our minds; and then indeed we shall see that to know just what we mean by classes and relations is at once a very important and a very complex undertaking.

Before we pass, however, to more novel and less abstract instances of conceptual forms, I must trouble you to take note of still another fundamental conceptual form which all science illustrates, which appears perfectly simple, and which is, nevertheless, really very complicated. I have spoken of classes of facts. I have also spoken [14] of the relations of facts. As you see, both the concept of a class and the concept of a relation thus seem to presuppose the concept of what you may call a single fact. If James is brother of John, then the relation here obtains between two individual men. If there were not some such individuals, there could not exist the relation.

Brotherhood is a fact in the world; but it is a peculiar sort of fact, which inevitably presupposes the existence of men who are brothers. Each of these men is a single individual. Precisely so, at least, as we usually conceive the matter, any class is a collection of single instances of objects which belong to the class. [15]

If there were no distinguishable instances that could belong to a class, this class would be a merely empty province, having value for thought only by its contrast with the classes that actually may contain single instances of objects which belong to these classes.

Both the concept of Class, then, and the concept of Relation, are conceptual forms which become useless for the purpose of thought without another conceptual form to help them out. This is the concept of the <u>single instance</u> which falls under a given class, or which, when it is taken [16] as the member of some pair, or of some other relational group, illustrates some relation. Now I shall use, as my name for any <u>single instances</u> of a fact which possesses a relation to other facts, or which falls under a given class, the name <u>Element</u>.

By an <u>element</u> I mean any one distinct object, a single fact, or anything that, for the purposes of thought, is for the time treated as a single fact. To speak in psychological terms, -- an element is whatever we fix by an act of our attention, and so, for the time being, distinguish or hold apart from other objects, while whatever variety [17] or diversity may be capable of coming to light within the single element, does not then attract our attention, or concern our thought. The concept of an element is, in consequence, a concept of a decidedly arbitrary application. Any object, however vast or complex, may be treated, in the course of a particular thinking process, as an element, for the purposes of just that process. A point is an element in ordinary geometry. But the straight line may be taken, instead of the point, as the geometrical element. And with reference to the class, <u>conic section</u>, or <u>triangle</u>, or <u>sphere</u>, [18] the corresponding single case of <u>this</u> conic section, or <u>triangle</u>, or <u>sphere</u>, is the element which forms the member of a given class, and <u>atom</u> is an ideal elementary constituent of matter. But the earth, or Jupiter, may be regarded as an element if you consider it merely as one of the members of the solar system. With reference to the collection of years which constitute the Nineteenth Century, any one year, say 1861, may be taken as a single element. And so one may go on, finding each time, as an element, in any class or group, whatever single [19] object may be distinguished, by the purposes of our thought, as for the time the constituents of the group. It is customary to say that classes and relational groups consist either of literal or of ideal individuals. If prefer the term element, to the term individual, as the name for what thought at any time distinguishes as the single constituent of any class or collection of objects of thought. And I do so because the name individual seems to imply, a metaphysical theory as to what an individual object is in itself, apart [20] from our own present thoughtful interest in it. But, as a fact, it is our present thoughtful interest which mainly determines, with respect to the objects once presented to our mental view, what we shall regard as, for the moment, our elementary objects. What is an element for one of our thoughtful attitudes, may appear as a collection from some point of view. What one train of thought views as simple, another investigation may regard as infinitely complex. A nation may be taken as an element when one is talking of a Congress of European powers, or of the chances of war in the Orient. [21] A single man may be an element to the employer of labor, or to the statistician, but a prodigiously vast complex to the physiologist or to the psychologist. And an element need not be, in itself, an individual. A color, a mathematical symbol, or even a relation, or a complex of relations, may be treated as an element for the purposes of a particular inquiry.

What is essential to the notion of an element is that an element is seized upon by our attention, is then, for the moment, treated as one, is distinguished from other elements, and is also grouped with them for purposes of fur- [22] ther study. When grouped with other elements, the single element enters into relations, and belongs to classes. And our whole thinking process makes use of these three conceptual forms: element, relation, class, whatever other concepts we may be employing. Because the term element is relative to our current purposes, any class may be taken as a whole, and treated as an element in a class of classes. Any relation may be taken apart from the terms which it relates, and may be treated as an element in a class of relations, or may be brought into relations [23] with other relations; and so we may proceed indefinitely.

II.

I turn then, for the moment, from these three most universal conceptual forms: Element, Class, and Relation, to more complex conceptual forms.

The text-books of logic are not accustomed to make sufficiently prominent what everybody nevertheless can readily observe for himself, namely that, in the most diverse sciences, and also in the most diverse extra-scientific thinking processes, we make use of rows, i.e. of ordered series of facts. The concept or conceptual [24] form of the row or ordered series is consequently one of the most widely used of the concepts of our thought. The numbers, which we mentioned at the last time, are a classic instance of a series. Our concept of a rhythm, that is of a periodic process, is another concept of certain types of series. The concept of series is thus more inclusive than that of either number or rhythm. It includes both of them as amongst its species. Nor do the ordinary numbers and the rhythms in the least exhaust the types of ordered series with which science and even daily life are acquainted. [25] The whole numbers form what maybe called a single, or in one sense, a simple series; for every whole number has its determinate place in that single ideal system of whole numbers which begins with unity, but which has no last term. On the other hand, various series may be interwoven, combined and complicated without limit, and yet so that the resultant complex system of many series may return a perfectly definable order. The pages of a work that is published in many volumes, form a compound series, where of the pages of a single volume may form a single relatively elementary, constituent. Here is an example of several series joined in one. But an ordered system may possess several dimensions. In each of these dimensions it will then give [26] us examples of various subordinate series. Space, as point-geometry conceives it, is a continuous system of points ordered in three dimensions. Any straight line is then conceived as a continuous one-dimensional series of points, selected from the tridimensional system of space. Time is generally conceived as a one dimensional series of instants. A moving body occupies a series of positions in space.

These are instances of series, and of serially ordered systems, which external experience often appears to exemplify to us, or even to force upon our thought. But we are often clearly aware that we deliberately construct certain artificial serial arrangements of real or of ideal objects [27] for the sake of aiding us either in finding these objects or in recalling them or in conceiving their various relations. Thus, if you put your books and papers in order, you do so by arranging them in certain chosen series, which may possess either one or more dimensions. A dictionary, a directory, a card catalogue, is a deliberately devised series of names, of words, or of titles. The term list stands for an artificially ordered one dimensional series. For purposes of conceiving the relations of color sensations, the psychologist groups them in series so that the resulting system forms a tridimensional color cone of [28] finite extent. Any map or diagram is an artificial serial arrangement of lines, surfaces, points, and marks, intended to portray the relations of objects. All statistical tables and diagrams depend upon highly artificial serial arrangements of groups of facts. The geologist arranges in series his conceived periods of the earth's history; the naturalist constructs his series of natural forms, and exploits them in his museums; the student of political science describes series of types of governments and depicts by serial devices the order which governs administrative processes. The clergy- [29] man or the lecturer presents his subject-matter arranged in serial order as firstly, secondly, thirdly. Every writer of a text-book has also to decide upon the serial order of his presentation of his subject. The dramatist uses serial devices as he arranges the order of his scenes and acts; the men on duty in the tower of a railway station deal with series of signals, of trains, and of apparatus for controlling switches; the switchboard of a telephone office is a system of orderly series.

Thus both the external facts and all the artificial devices of our thought constantly exemplify the [30] concept of series. It is strange that, until quite recently, the general lyrical aspects of the concept of what constitutes a series have been comparatively neglected. While classification, in certain of its formal aspects, has been extensively discussed in the text-books of logic, serial order without which classification would have little value for our thought, has merely been named, but has otherwise been far too much neglected by the logicians, until within the last fifty years. Kant's Critique of Pure Reason hardly more than occasionally mentions the concept of serial order. This concept has more of a place in Hegel's Logic, yet no adequate discussion. The psychologist Herbart made the concept prominent in his discussion of the concepts of space and [31] of time. But it has been the interests of modern mathematical inquiry which have made the logical discussion of the types of serial order a prominent issue in recent thought. Georg Cantor, Dedekind, Schroeder, Mr. Charles Peirce, Bertrand Russell, are prominent recent investigators of the concept. The best summary of the modern discussions of the whole question in English is to be found in Mr. Bertrand Russell's recent work on the Principles of Mathematics.

An ordered series is a peculiar sort of class of objects, whose members are subject to certain characteristic relations. What these relations are, we shall state, in the most general way quite soon. For the moment, I merely want to call your attention to [32] general truth that all orderly systems of facts, whether these systems are forced upon our notice by our external experience, or are artificially invented to meet the needs of our science or of our arts, get their order from the fact that they consist of various ordered series of real or of ideal objects. The concept of a row or series of objects is therefore a concept of simply universal significance for every coherent thinking-process.

III.

But the process of arranging objects in series, or of taking account of serial arrangements, is very frequently connected, in our thoughtful processes, with another and a decidedly contrasting process. This latter process [33] leads to a conceptual form whose wide usefulness is as obvious, upon mention, as the comparative neglect of logicians to consider its nature and types is notable, and regrettable.

When an apothecary has a collection of weights which he uses for his scales, these weights form an ordered series. But when he has to weigh out goods, and when he accordingly puts certain weights into one scale pan, and then pours a certain powder, or lays any set of objects, upon the other scale-pan, -- the goods that he thus weighs are not put into that same ordered series which consists of his [34] standard weights, so as to become terms of that series, but are found, in the end, to correspond in weight, to be on a level with one of his standard weights, or with the sum of several weights. When a chronologist compares the dates of Egyptian and Babylonian, or of Greek and Roman history, the dates compared are, in the end, brought to levels with one another, so far as they are the dates of contemporaneous events. When a naturalist or a lithologist takes a cross-section of a preparation or of a rock, he does not do so in order to find in what various places in the same ordered series, certain structures stand; but rather in order to find what objects [35] are visible upon the same level, namely upon the level which his cross-section establishes. In other words, he thus finds out, not what comes before or after in a series, but rather what his side by side at a certain point, or on a certain level, in a given series.

Now the thoughtful operation thus exemplified, the operation of what I may call <u>taking levels</u>, is an operation of the utmost importance for thought. It is a sort of thinking that we exemplify whenever we say: “This object equals that object,” in weight, length, mass, temperature, [36] value, or other magnitude. It is the kind of thinking-process which we go through when we compare various ordered series of objects and say: “The object a, in the series s, corresponds to the object b, in the series t"; as for instance the event a in some "series a belonging to Roman history corresponds in time to, is contemporaneous with, the event b in Greek history. Or we take a level when we contrast any group of coexisting facts with a series of successive groups of facts, or when we observe that two points on the earth's surface are upon the same [37] parallel of latitude, or have the same mean temperature in February; or when we learn from a geologist that a certain strata or certain fossils belong to the same geological horizon. In the physics of a field of force equipotential lines and surfaces are instances of levels. In the history of law, of politics, and social life, the assertion of the equal rights of all men is the effort to place all men, in certain respects, not in any serial order, but upon a level. In the practical arrangement of both material and social objects, levels are quite as important as series. equality and precedence are the two correlated [38] practical problems of all our social relations. Precedence, in social affairs, has to do with ordered series; equality has to do with social levels. As to material objects, we order them in series; but we also arrange them so that various books are on the same high shelf, various rooms on the same floor of a building, various places are at nearly equal distances from certain central points, and so on. Now placing objects on the same level does not, of itself set them into any ordered series. Thus to know that many objects are of equal weights, is not to set them into any series. But the process of taking levels serves to correlate many series, and is itself an indispensable correlate of the series-building process. [39]

Of what vast and wide spread importance the conceptual process of taking levels is, we are further reminded by the fact that all the exact sciences constantly use equations, so that the equation, which is an instance of the process of taking levels, is found wherever thought most succeeds in its work. We are also reminded of the same truth by the fact that the doctrine of the Conservation of Energy is due to certain observations which I should describe as the taking of levels, and that the same is true, as we shall later see, regarding every conception of physical constants and of invariant laws of nature. [40]

That in thus classifying together the processes which I call the taking of levels, we have not been merely throwing into one group a miscellaneous collection of conceptual forms, I shall soon be able to show you. Ordered series, as we have already said, are characterized by the prevalence in them of a peculiar type of relations. Levels, or groups of objects that are upon the same level, are logically distinguished by the prevalence in their case of a very different type of relations, which is characteristic.

IV.

But in addition to the conceptual forms so far discussed, we find in the field of science still another and very important group of concepts which are, indeed, closely related to the foregoing, but are also to be distinguished from them. These are the concepts we may call the general "Transformations." They are of these classes, which I shall at once exemplify.

(1) The world of our experience presents to our attention at all times a constant series of Changes. Any one of the changes that occur may attract our attention either for theoretical or for practical purposes. For a change is as much an object of experience as a thing, and we are as much concerned to notice what happens as to observe what at any time stands before us as a stationary fact. Now Changes occur throughout the whole range of the mental world. In the physical world change does not appear to be so pervasive, and we have a more obvious contrast between the permanent and the changing, a contrast upon which our conception of natural objects very largely depends. All changes so far as we notice them, fall into types. They may be classified. They differ as to their practically important or their theoretically describable characteristics. We therefore form throughout the whole range of a science conceptions of Types of Change.

(2) But we are not content to let the changes of the world pass before us as they happen to do. We are active beings. We interfere with the course of change. We make changes to suit our own purposes. And we conceive these types of change whenever we form a plan of action or define the processes of any art. We consequently need a name which shall include the concepts of those types of change which are presented to us by the course of nature, apart from our interference and those types of change which we regard as subject to our will, and which we produce or not according as we [42] desire. The general name "Transformation," widely used in modern mathematical science, seems to be the suitable name to represent when is common to both these types of change [illegible] which occur apart from our will and to those types of change which are more or less completely due to our interference.

But (3), the concept of transformation needs still one further extension. Just as we are not content to let natural changes occur apart from our interference, and constantly interfere with the course of nature, so, on the other hand, we are not content merely to observe and [illegible] changes that literally take place in the external world. We take great interest in conceiving Ideal Types of change, which either may under certain circumstances occur in the outer world, or which in some cases may be intended to remain wholly ideal. In processes, namely, of calculation, of computation, of reasoning about ideal objects, we are frequently concerned in defining certain types of change which we bring about wholly in the realm of our ideas. Thus, for instance, we may <u>substitute</u> one idea for another at any given stage in an investigation. As we may combine certain ideas, and, for the sake of defining the result of this combination, we may then substitute for the combination some new idea. Transformations that occur in this way, in the world of our ideas, may also receive the name of Ideal Operations.

A classic instance of transformations of this kind is furnished by the well-known mathematical operations – addition, multiplication, division, subtraction. Operations of this sort are transformations occurring in the realm of our ideas. They consist in certain substitutions of ideas such as those of sums and products, for certain other groups of ideas. These transformations occur according to the definite rules of the science in question. Any process of reasoning, whereby [43] we proceed from the premises of an argument to the conclusions of this argument is a process of ideal transformation which is covered by the rules of logic.

Thus, in every science, we are obliged to have conceptual forms that especially have to do with transformations, where these transformations may be (1) the changes that the so-called external world presents to our experience apart from our interference, on (2) the changes that we deliberately introduce into this world for the sake of our practical purposes, and for the sake of scientific experiments; and (3), the changes in question may be purely ideal transformations such as occur in a computation or in a process of reasoning.

Concepts of transformations, in our sciences, stand in the most intimate relation to the other types of concepts that we have already exemplified. Thus, in general, transformations that we consider in any definite scientific process are likely to form a more or less extended series. The same interest that leads us everywhere to carry out the process which we have called in the foregoing the “taking of a level,” may lead us to compare various transformations together and to regard their results as, from a certain point of view, equivalent. And so concepts of what I may call “levels of transformation” play a great part in all the computing sciences. But if we thus consider, series and levels of transformations, we must of course make use, in dealing with transformations, of those still more fundamental scientific concepts that we mentioned at the outset. For transformations, or types of change, natural and artificial, real and ideal, fall into certain definable classes or can stand to one another in certain definable relationships. [44]

V.

We have thus taken our first general survey of the fundamental types of scientific concepts. Classes, relations, ordered series, levels, transformations: these constitute the fundamental types of concepts which you will find useful in the most various sciences and in the most various branches of our commonsense activity. That in classifying thus the concepts of which our thought makes use we have not been making merely arbitrary distinctions or chance collections of types of concepts will, I hope, be evident from the illustrations already given. What we next need to do is to become somewhat more clearly acquainted with the characteristic structure of each of these various types of conceptual forms. In order to do so we must return very naturally to those simplest forms of which I spoke at the outset of this lecture.

Let me then next ask the question, "What do we mean by a class of objects?" "What type of concept, what conceptual form is necessary to characterize what we mean by a class?" A very common answer to this question you will find in many text-books and in many popular discussions. This answer says that a class means a group of like objects: that to classify objects is to put together like objects and to separate unlike objects. This account is, of course, not without its significance, but it is extremely inadequate to characterize the essential logical process which takes place when we classify. As a fact, there is no group of objects so unlike that we may not for a given purpose form a class of those objects. And where we can make any distinctions at all, no matter how closely various objects or elements resemble one another, we can classify. The perception of difference is a necessary [45] preliminary to classification. But it is so merely because, unless we can discriminate, we cannot observe or conceive of the elements that we are to classify. But if I have once distinguished elements, I can then proceed to classify without being at the mercy of a mere following of observed likenesses or unlikenesses of things. How trivial it is to say that a class of objects consists of like things while unlike are as such put in different classes, appears if we remember that the term "unlike" is a general term, and consequently, whenever used with a definite meaning must name a class of objects. To say of certain things that they are unlike on another or unlike certain other objects, in certain respects, is at once to name a basis for classification; just as it is a basis of classification if you observe of certain mean that each one of them is a brother to certain of the others. As brothers constitute a class, so unlike things constitute a class, if for any definite reason you are interested in pointing out a specific unlikeness. It is, of course, also true that the members of any class are in some respects like one another. But the same could be said of the members of any two different classes. Likeness and unlikeness, taken in their most general sense, are names for relationships that you can illustrate all over the universe. While these relations unquestionably guide our activities in classifying, it is useless to endeavor to define classification solely in terms of them.

One comes nearer a definition of what one means by a class of objects when one remembers that any definite class of objects always has, as its attendant in our minds, another conceived class [46] of objects, which is what we call negative to the first class. Thus if you conceive of men as a class, you contrast them in mind with persons who are not men; and while of course it is true that the mean and the not-men are in certain notable respects unlike, and not-men in that there are certain judgments which you hold to be true of every man, and which you hold to be false, or untrue, of every being who is not a man. In other words, classification depends upon a certain distinction between true and untrue, positive and negative judgments. I can observe likeness and unlikeness without consciously judging. But I cannot classify unless I am ready to judge. The logical prerequisites of the process of classification are these!—I must first observe or conceive of some element or other, as for instance this mom [?], or this number I must then have some tendency to make a judgment about this elements. Now a judgment, an act of assertion, is in certain respects like any other voluntary act. That is, you either perform this act or you do not perform it. If you consider it or think it over, you either accept it or reject it. When I accept a judgment, I call it true.

When I reject a judgment, I deny it or call it false. And the distinction between the falsity and the truth of a judgment, the assertion and denial of a proposition, this is a distinction which for the moment I do not here attempt to analyze. But unless you make that distinction, you can never learn to classify.

Now in advance of defining, let us say the particular class of objects known as men, I can still judge about this object before me. I may observe, namely, that something is true of this object, that certain predicates can be attached to this object. No matter, for the moment, what these predicates are. Now repeated experiences [47] of various elements or objects have led me to take note of the fact that there are some in whose presence I assert that a given judgment is true. For instance, there may be various objects to any one of which I learn to attach a predicate such as we find applicable to human beings. Thus I observe not only that some single element arouses in me the tendency to judge thus or thus, arouse in me the tendency to judge them all in the same way. Hereupon, in a fashion of the being who is able to form the idea of a class of objects, one proceeds to define to one's self a certain totality of objects. This totality remains an ideal collection of objects. It is the collection of objects any one of which I shall judge in this same way. That is, it is the collection of objects to any one of which I shall attach such a predicate as I first attached to some individual, thus man means the totality of being to any of whom I shall attach whatever predicate I choose to regard as the predicate that defines what is essential to constitute a human being. For example I might have noticed of the individual man that he could speak, or laugh, or reason. I could then proceed in my mind to form the conception of an ideal collection of objects, of any one of which the assertion he can laugh or speak or reason would be true. I may then proceed to notice that, if any object did not belong to this collection, some one of these judgments would not be true of that object. I may decide conversely that if of any object some one of these predicates was not true, such an object should not belong to my collection called "man". The result of all these processes would be so, so far as I could make the process exact and conscious, a definition of the class “man”. If the definitions “rational,” “capable of speech,” “capable of laughter,” or any other collection of predicates, have been chosen to define characters essential to man, the class “man” ideally consists of all objects, to any one of which these predicates truly apply. The [48] class “man” excludes all elements, objects, things, to any one of which these predicates, or any one of them, shall not apply.

I could sum up the whole process of classifying in still another formal way. Suppose I have found an elements “a.” Suppose I have learned to assert of this element some predicate “P”; so that I hold the judgment “a is P” to be a true judgment. Suppose that I now in general conceive of a sort of skeleton of a proposition, a form which a possible assertion might take. Let me express this mere form by the words “x is P,” where for a moment “x” does not stand for any one element but for any objects whatever, within the range of the objects that I propose to take into account at all. Then I conceive of a class of objects if I say: “All those objects ‘x’, of any one of which the assertion ‘x is P’ would be true, shall be grouped together as a class. Every objects of which the assertion “x is P” is false shall be excluded from this class. By some such test, by finding some such propositional form true or false of any one of a collection of objects, we learn to distinguish between an object that belongs to a class and an object that does not belong to a class. The essential thing to remember is that classification depends not merely upon the perception of likeness or unlikeness, but upon the acceptance or rejection of a certain judgment, as applying or as not applying to any one of a group of objects. The judgment, whatever it is that we use as the test for distinguishing between the members of a class and the objects which are not members of a class, we call our definition of this class. Where there is no definition, there is no logical class. A definition is an assertion that must be either true or false of the [49] objects that we propose to classify in accordance with this definition. If the definition does not apply to certain objects at all. Or becomes meaningless if you try to apply it, then those objects cannot be classified in accordance with this definition. The class of objects consists of all those elements or things such that this definition is true of any one of them. Since we can conceive objects, and in general can find objects of which the definition is not true, we regard these objects as forming a class, which we call negative to the class first in question. And thus a group of unlike objects may form a class, if the possession of a certain definable unlikeness can be truly asserted of every one of those objects, or of the members of every pair of those objects that you can form. Again, the defining predicate, proposition, or skeleton of a proposition, in terms of which we define a class, may make no assertion whatever about likeness or unlikeness, except in so far as such an assertion is inevitably implied in every judgment whatever it be. For I can classify together all the objects that I chance to see during a certain walk, or all the objects, whatever they may be, whose names occur anywhere in Gibbon's History of Rome, or in a certain auctioneer's catalogue. It is true that scientific classification generally requires us to take certain decidedly significant propositions as our definitions of classes; and when we define we usually take account of the value of our definitions as pieces of information. But the essential logical characters of a class are those now set forth. In order to classify you must use so deliberately chosen judgment as the test whether or no this or this object does or does not belong to a given class. [50]

VI.

I turn from these elementary considerations about classes, to a somewhat more important consideration about the nature of Relations. The relations which are useful to us in the work of our thought, seem at first sight to be of endlessly numerous types. Everywhere, when you pass to any new science, or to any new experience, you seem to meet with new relations. To reduce the world of relations to a few simple types would seem, therefore, to be a hopeless enterprise. The study of relations would seem to be identical with the whole task in which all sciences together are engaged. Yet in the modern study of the logic of relations, it has turned out that a certain few and special types of relations are of such enormous and universal importance in the whole field of definite thinking that a consideration of these few types of relations is already sufficient to throw a very important light upon the interconnection of all our thinking processes. Let me point out what the fundamentally distinct types of relations are. Then we can at once be sided to understand those concepts, of ordered series, of levels, and of transformations, which we have already seen to be of such significant in science.

Let me first attempt to define what one means in general by a relation. I say that a man is somebody's brother or debtor or friend. When I assert such things, I point out this man's relations. Evidently I attach a predicate to the man of whom I assert the relation. But obviously this is a predicate that this man could not possess, if he retained his physical existence but were otherwise alone in the world. If there were no other man in existence, this man could not be brother, debtor, friend. So a relation is a characteristic that belongs to an object not when you take it alone by itself, but when you take it as a member of a group of objects. In other words, if you meet with a character which belongs to an [51] object so long as it is a member of a certain group of objects, but which this object loses if its membership in that group ceases, then this character must either be, or else must directly result from, a relation in which this object stands to other members of the group of objects. Thus any character that a man loses, if he dissolves a certain partnership or retires from a certain club or abandons his citizenship in a certain nation, is a character which must, either consist in his relations to certain objects or must directly result from those relations. We shall therefore not go far wrong if we say that <u>a relation is a character which belongs to an object, in so far as that object belongs to a group of objects.</u>

The group of objects with respect to which a given element may be said to possess a certain relation, may as we have said be a very small group. It is frequently a <u>pair</u> of objects. Thus if "a" and "b" walk together, they have the relation of being companions. And when we are asked what the relations of certain objects are, we find a strong tendency in minds to consider these objects merely in pairs. If I am asked, “In what relation does ‘a’ stand to other objects,” I often answer the question by first asking the question, “In what relation does ‘a’ stand to ‘b’?, In what relation does it stand to C?” and so on. It is therefore customary to say that a relation is some kind of tie or connection that holds between two objects, or in our language, is a character that belongs to a member of a pair of objects. But this limitation of the conception of a relation is an unfair one. It is true that when I say I am fellow-citizen to every citizen of the United States, I assert a relation, which I am likely to test, by comparing myself first with one citizen, then with another, and so on, [52] so that the relation comes to my consciousness when I consider pairs of objects. But there are certain relations which I can have, which I possess only so far as I belong not to a pair of objects, but to a group of three, four, or more objects. It is to the credit of Mr. Charles Pierce that the logic of relations of these more complex sorts have been, in this country, considerably discussed. As Mr. Russell says, in his discussion of the subject of relations, the philosophers have been too much disposed to reject, or at least neglect, these more complex relations. On the other hand, mathematicians have gradually come to give them more and more prominence. And as a fact, commonsense more or less unconsciously recognizes them all the time.

Consider, instance, what meaning is expressed by a sentence in which occurs a verb that has a direct and an indirect object. Thus take the statement, “a gives b to c”- as for instance, “John gives James an apple.” John, James and the apple here stand in a set of relationships, each one of which is definable only in so far as you consider all three objects. John is related to the apple as the giver of it. But what mean by his giving the apple only appears in case you at least indicate to whom he gives it. The receiver of the apple has again a position that can only be defined in terms of his relation both to the giver and to the apple. And the apple is obviously in a relation that is definable only through the transfer of possession from the giver to the receiver. And so this group of three objects is such as to define several three-cornered relations. To pass at once to a more complex case, the players on a baseball nine constitute a group of persons who have certain definite inter-relations. The pitcher has relations to [53] the catcher, to the various base-men, and so on. These relations may determine the whole work done during the practice of the nine, and will certainly appear in the course of its actual play. But the relations of each player to each one of the others are unquestionably such as are determined by their common relations to the other players. A complete understanding of what you mean by calling a man the pitcher of the nine, can therefore only be obtained if you take account of all the players of that nine. Here is what we might call a many-cornered, or polygonal system of relations.

For our present purposes it is enough merely to take note in this way of the fact that relations differ according to the size of the group of objects within which they hold, and further to say that relations which hold between members of a pair of objects are technically "dyadic" relations, while relations that hold of the members of a group of three objects are technically called "triadic" relations. Dyadic relations are expressed in language by verbs that have a direct object and no indirect one, and sometimes by prepositions used in conjunction with the verb "to be." Dyadic relations are also frequently names, or indicated, by such nouns as "friend," "brother," or other relational term. Triadic relations are especially expressed as just pointed out by sentences in which a verb with a direct and with an indirect object is used.

A very important and fundamental logical question is that as to whether dyadic relations are any more fundamental, than more complex types of relations. It is customary to hold that they are the fundamental and characteristic types of relation. It is for that reason that some have neglected altogether or have rejected the more complex relational types. But considerations at which we may have time to glance before I finish this course of [54] lectures, tend to suggest that the triadic relations are really more important for our thought than dyadic relations, and that they are logically relations of the more fundamental type; at least so long as we consider certain of the most important interests of our thought. But this will come into sight only further on in our work. For the moment I will consider dyadic relations as if they were the most obvious and natural relations. And I will proceed to point out the most important classes or types into which they fall.

VII.

A dyadic relation such as “friend” or “brother” holds, we have said, between the members of a pair of objects. But if we consider a pair of objects, we observe at once that it makes a great deal of difference to our thought in what sequence we take these objects, as we consider them. Thus if I say that “a” is greater than “b,” the sequence in which I take “a” and “b” determines whether this proposition is true or false, in case “a” and “b” are quantities or magnitudes that are to be compared. If “a” is greater than “b,” it is not true that “b” is greater than “a.” On the contrary, so soon as I read the truth in question in respect of reverse sequence of the two objects, I must say that “b” is less than “a.” Each of these two propositions, “a is a greater than b,” and “b is less than a,” follows from the other proposition. Each is another way of reading the same truth. It seems natural to express this by saying that the relation of “a” to “b” is distinct, and may be very different from the relation of “b” to “a.” Numerous pairs of propositions, such as the assertions, “a is b’s debtor,” “b is a’s creditor”: “a is to the right of b,” “b is to the left of a”: - such pairs of propositions, I say, will suggest to us how important for the statement of a relation is the sequence in which we read [55] its terms, while the relation of “a” to “b” in general determines what the relation of “b” to “a” is.

This fundamental fact that the sequence in which we read the terms of a dyadic relation determines the way in which you must express this relation, holds true of every dyadic relation. But, nevertheless, we meet with the fact, that in certain cases, the relation in which “a” stands to “b” is actually the same as the relation in which “b” stands to “a”. In other words, we may observe that while the propositions “a is related thus or thus to b” is different from the proposition “b is related thus and thus to s," nevertheless it may be actually true that "a" stands in a certain relation "R" to "b", while at the same time "b" stands in this same relation "R" to "a". Now when this is true of a given relation, we call the relation "symmetrical." In other words, a "symmetrical" relation is one which holds between two objects in whichever sequence you may choose to read them. A familiar case of a symmetrical relation is the relation of equality. If "a" is equal to "b", "b" is equal to "a". A symmetrical family relation, if we are considering a group containing men alone, is the relation of "brother." That is, if "a" is "b's" brother, "b" is "a's" brother so long as you deal with men only. In English the relation "cousin" is treated as a symmetrical relation, whether we deal with men or with women.

On the other hand, such relations as those of "Greater and less," "father and son," "debtor and creditor," are technically called "non-symmetrical", or "asymmetrical" relations. If the relation "R" of "a" to "b" is an asymmetrical relation, then "b" has to "a" a relation different from the relation "R", which is [56] called the "converse" of "R". Thus if "a" is greater than "b", "b" is less than "a". If "a" is father of "b", "b" is son of "a" or daughter of "a" as the case may be. The fundamental importance of the distinction between symmetrical and asymmetrical relations, for all purposes of scientific thinking, will soon appear. It remains here to be said that there are a few relations of importance which may be called sometimes symmetrical and sometimes asymmetrical. In the usual way of naming family relations, the relation of a brother to one to whom he is brother is of this type. If you are dealing with men, the relation "brother" is symmetrical. But if you are considering family relationships between men and women, then when "a" is "b's" brother, "b" may be "a's" sister, and hence the relation of "brother" is sometimes asymmetrical. A very important relation of this type, is the relation that plays so great a part in all our processes of reasoning, the dyadic relation which we may call the "illative" relation, - the relation which permits us to infer or which holds between the premises and the conclusion of an argument, or between the antecedent and the consequent of a mathematical proposition. If I know of two propositions “a” and “b”, that “a” implies “b”, or that “b” follows from “a”, I may not conclude that “b” implies “a” or that “a” follows from “b”, although this may be the case. The relation of implication, the illative relation, is therefore sometimes symmetrical and sometimes not. To know that in a given case it is symmetrical, is to know something that may be of great importance for our reasoning processes. When in the geometry textbooks the converse of a theorem is proved, the result is that we are able to show that a particular inference is a symmetrical one. But the process of proving this in a given case may be difficult. [57]

VIII.

So much for the fundamental distinction between symmetrical and non-symmetrical relations. But next I come to another distinction of equally fundamental importance. A given dyadic relation may hold true in various pairs of objects. Thus we are considering four men, “a” may be brother of “b”, and “c” may be brother of “d”; so that the same relation holds in different pairs. But sometimes it may happen that we consider two pairs of objects, while these pairs also have a common member. Thus “a” may be brother of “b”, and “b” may be brother of “c”. But in this case, as we know from the nature of the relation of brotherhood, if we are considering a group of three objects and we find these two propositions true, we can proceed to assert “a” is brother of “c”. In other words, we can draw a certain important inference, from observing a certain dyadic relation to be present in two different pairs that still have a common member. The same kind of inference may be drawn in case we are dealing with the relation of equality. If “a” is equal to “b”, and “b” is equal to “c”, “a” is equal to “c”, or as the ordinary axiom states it, things equal to the same thing are equal to each other. That we can draw this inference is, however, a very important characteristic of the relation of equality. Now by no means every relation permits us to draw such inferences. If “b” comes nest after “a”, and “c” comes next after “b” in a row of objects, it does not follow that “c” comes next after “a”. On the contrary, such a proposition in the ordinary meaning of the relation “next” after would be under the circumstances false, the relation “next after” differing, then, from such a relation as the relation of equality in not permitting the [58] inference of the type mentioned. Again, if “a” is different from “b” and “b” is different from “c”, it does not follow that “a” and “c” are different each from the other, although this may be true. If “a” is “b’s” debtor, and “b” is “c’s” debtor, the business relations amongst the three may be such as to secure an arrangement whereby “a” appears as “c’s” debtor in consequence of the first pair of relations. But this result does not always practically hold. That is, it may not always be possible for “c” to collect from “a” directly, as a debt due to “c”, that which “a” owes to “b” when “b” is “c’s” debtor. We see, then, that certain relations permit, and that certain relations do not permit, a type of inference which is of the utmost importance for many scientific.

We consequently distinguish two classes of relations, in the world of dyadic relations. We say, first, that a dyadic relation “R” maybe transitive. And by a transitive relation R we mean a relation such that if “a” stands in the relation “R” to “b” and “b” stands in the relation “R” to “c”, then “a” stands in the relation “R” to “c”. On the other hand, a relation “Q” such that if “a” stands in the relation “Q” to “b” and “b” stands in the relation “Q” to “c”, “a” cannot stand in the relation “Q” to “c”, is a wholly intransitive relation. An example of such a relation is the relation of father to son. If “a” is father of “b” and “b” is father of “c”, then “a” cannot be father of “c” but stands to “c” in another relation, that of grandfather. But there are certain relations which are sometimes transitive and sometimes intransitive. An example of this we see in the relation “different from.” Thus if “a” is different from “b” and “b” is different from “c”, “a” may or may not be different from “c” according to “a”.

We have thus divided dyadic relationships in two different [59] ways into symmetrical or non-symmetrical or asymmetrical relationships, and into transitive and intransitive relationships. In the case of each classification we meet with some relations that are in some instances of one and in some instances of the other of the two classes distinguished, that is, we meet with some relations that are sometimes symmetrical and sometimes non-symmetrical, and also with relations that are sometimes transitive and sometimes intransitive. These intermediate types of relations are usually of less significance, at least for the purposes of all exact sciences; although we indeed have one relation which is sometimes symmetrical and sometimes not, namely the "illative" relation, and which is of great importance in all science. But now, having distinguished the two different classifications, we see that the two classifications are in a large measure independent of each other. A relation may be symmetrical and transitive it may be either of the symmetrical or non-symmetrical type. We thus get four classes of relations, which are of fundamental importance.

Having made these rather wearisome but indispensable distinctions, we may now make an assertion which I shall attempt at the next time to illustrate and to apply in a way whose importance must justify the apparent formalism and abstractness of the distinction in question. I assert, then, as I close, that those conceptual forms which we have called "ordered series" depend altogether for their existence and for their significance upon the presence of transitive, non-symmetrical relations, which obtain between members of pairs of objects that you find belonging to a series. That is, I assert that those conceptual forms which we have called ordered series depend upon the presence of [60] some such relation as "before and after", and this relation may be in general defined as a non-symmetrical, but transitive relation. On the other hand, what we have called a "level" is characterized by the presence of a certain relation such as "equality," which obtains between the members of any pair of objects belonging to the same level; Equality however, is a transitive symmetrical relation. Therefore, as we shall see, upon the distinction between the symmetrical transitive relations and the non-symmetrical transitive relations, the most fundamental distinctions between the concepts of the various sciences depend. The entire business of thought may be defined as a process which everywhere includes establishing systems of relations and then distinguishing transitive symmetrical relations, from transitive non-symmetrical relations. Both these types are of course frequently distinguished from the intransitive relations, whether the latter be symmetrical or non-symmetrical. But while the intransitive relations are of great significance in enabling us to deal with the details of the facts of our world, the transitive relations, symmetrical and non-symmetrical, are of significance to us in establishing all the larger connections, all the more extended series, all the more widely significant comparisons and levels and equations, and laws, of the world with which our thought deals. At the next time I shall show, then, in more detail how these distinctions may be applied to the conceptual types that we already distinguished.