The Columbia University Lectures

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Copied and arranged by Scott Pratt

Columbia Lectures

1904

Programme of Lectures

(Columbia Course)

Some Characteristics of the Thinking Process.

First Lecture:- Introduction: The Comparative Study of the Concepts of Science. Examples of Concepts useful in widely sundered regions of Inquiry. Problem as to the reason for this usefulness: Is ~~the~~ this reason to be found in the nature of things, or in the nature of the thinking process? Arguments for both views.

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Second Lecture:— General Survey of certain concepts that are of fundamental importance in science:—

(1) Classes and the process of Classification.

(2) Relations and their types ~~classification~~.

(3) Ordinal Concepts and ordered Series.

Third Lecture: The same topic continued.

(4) Concepts of Transformations:—

a. Concepts of External or Actual Transformations

b. Concepts of Ideal Transformations:— the "Operations" of the Exact Sciences,

(5) Concepts of Levels:—

a. Types of Equality and Equivalence

b. Concepts of Invariants or Laws,

(3)

Fourth Lecture:— Applications of the foregoing survey to ~~the [illegible]~~ various special problems:— Intensive and Extensive Magnitudes; the problem of Descriptive Science as the definition of Manifolds, and their adjustment to the varieties of fact; the search for an universally applicable ~~[illegible]~~ Manifold.

Fifth Lecture:— Philosophical Considerations suggested by the foregoing survey. The problem of the Categories. Three ~~[illegible]~~ views of the work of thought: Realism, Pragmatism, and Idealistic Absolutism.

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Columbia Lectures (1904)

Some Characteristics of the Thinking Process.

Lecture I. Introduction.

The study of Logic is usually regarded as a dreary task. The beginner in philosophy takes his course in ele:mentary Logic as a sort of necessary evil; — a price that one sometimes has to pay for the oppor:tunity to consider, in later courses, genuinely interesting themes. In a generation that is now past, John Stuart Mill's Logic did indeed give the subject of the Logic of Induction a fascination for many

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readers to whom the traditional Logic
of the text=books meant little or nothing.
But Mill is no longer the living influence
that he was. There is at present no substitute
for his book which can fill the place that
in his day, his Logic occupied. The result
is that students of philosophy too often
fail to find Logic meaning anything
to them but a task which a dead tradition
has made a more or less necessary part of
their academic undertakings, but which they would
fain avoid if they could, since it has nothing
to do with their progress as thinkers.
Yet this bad reputation of
Logic is founded upon a mere

MS. torn before mounting

(3)

...ance of the true scope of the subject
...ignorance for which the text=books are indeed largely responsible.
...ly regarded, Logic is concerned not
...with vital, but with very progressive
...ries. Scattered through the whole
...ture of science there are, today,
... various remarks, queries, and
...ches, of a logical nature, which have in many cases
...ng directly from the needs of specialists,
... generally progressive inquirers. These men,
...appling with the concrete problems of their
own sciences, have been led, in the very midst
of their conflict with the baffling facts, to
reflect upon the meaning of their thoughtful
methods. They have seen that, since they
wanted rightly to conceive their data

they were obliged to understand their own processes of conception. They have observed that they could organize rationally their materials, only in case they took conscious account of the nature of rational organization. Now all such contributions to the theory of sound thinking, whenever they are successful, constitute advances in Logic. While the text-books that are formally devoted to Logic are still, for the most part, lamentably inadequate and antiquated accounts of the ideals and methods which guide thinkers, the general advance of logical doctrine, in the form of the scattered but effective contributions of various workers, is constant and is still

of interest.

It is my purpose, in the following brief course of lectures, to give a few illustrations of certain problems of recent logical inquiry. I speak as one who am myself only a very imperfect inquirer in this branch of research. I have no very highly finished results to report. I chiefly want to win for this field of study new devotees. As I am to address a company of students, to whom the rather widely scattered literature of the subject of recent logical investigation has probably not been very accessible, I shall make little attempt to be original. I shall be content, during most of these brief studies, simply to report some of the less popularly known recent investigations into certain topics of logical interest.

I.

Let me first describe, however, a little more fully, the scope and limits of these lectures.

Logic was traditionally divided into three parts, which treated, respectively, of the three socalled divisions of the thinking process, namely (taken in their traditional order) Conception, Judgment, and Reasoning. It has become customary, in modern treatises, to point out that, if these divisions of the thinking process are to be kept apart at all, their traditional order needs, some change. For if any one of the three is to be regarded as the primary and fundamental

operation upon which thinking is based, then Judgment, rather than Conception, is to be entitled to the first place. It is true that whoever judges, inevitably possesses (at the moment when he judges) some conceptions (or some grade or value). But, on the other hand, every new conception is reached, either as the outcome of pro= or as the accompaniment of processes of judgment. Our earliest conceptions, as I think that we may say, first came to our consciousness, became our actual possessions, at the moment when first we judged. Our later conceptions receive, at each stage of our advancing thought, their first formulations by means of judgments. The traditional doctrine of the text=books defines a judgment as a synthesis of (previously existing) concepts. It would be nearer the truth to say that a judgment is a process that is busied either in the building of new concepts or in the modification of older concepts for the sake of adjusting them to new experiences. Neither account is an adequate statement of what happens when we judge. And we are aided in understanding the matter by adding that an already formed concept, when it comes to our consciousness, constitutes a sort of plan of action, an epitome of a series of processes whereby, as we believe, we may, if we choose, portray or construct objects; while a judgment is an actual deed, a construction or portrayal of an object, joined with a consciousness that this construction or portrayal is what it ought to be. But we learn to form plans of action by first acting. Hence judgment is a necessary means for forming new concepts.

P. However one defines the relations between conception and judgment, the fact remains that any scientific

conception of importance and of any high degree of elaboration, is a resultant of a great number of previous processes of judgment, and also of actual reasoning. As for reasoning, it is a process of judging about judgments, and about their relations and their meanings. And as both simple judgments and inferences (i.e., judgments about judgments), enter into the processes whereby all our more elaborate conceptions are formed, there would be much to say for an order of logical exposition that followed the plan of discussing, first the judgment, then the process of reasoning, and thirdly, the process of conception.

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This way would ~~have~~ (indeed also have) its faults. For all our higher and more complex processes of judgment and reasoning, ~~[illegible]~~ presuppose simpler concepts that have been built up through or in various previous processes and series of judgments and reasoning. But the fault of the traditional procedure (of the textbooks ~~[illegible]~~) ~~which goes~~ is a much greater fault. For it gives ~~[illegible]~~ to the elementary student the impression that we first form finished concepts, then unite them in pairs when we judge, and then unite the judgments into syllogisms, — ~~[illegible]~~ Really judgment is the very life of the thinking process. We can form no new conceptions except by means of judgments. Inferences are judgments about the relations and results of judgments.

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Conceptions, in all their higher forms, are products and more or less epitomized results of previous processes of judgment and reasoning. Hence the theory of the concept is really the most advanced and elaborate of these three divisions of Logic.

Now in this ~~brief~~ little course of lectures, I shall devote myself, in the main, to ~~[illegible]~~ this most advanced of the three traditional divisions of Logic, namely to Conception. ~~with you and, I propose to consider,~~ a few ~~[illegible]~~ of the most notable types of scientific Concepts (and to draw some conclusions from this study). ~~That is, after a brief preliminary~~ While I shall need to introduce some brief discussion of the processes of judgment and reasoning (by which such concepts get formed), I propose to ~~make a~~ devote myself, for the greater part of these lectures, to a comparative study of some ~~[illegible]~~ of the most remarkable ~~[illegible]~~ conceptual structures, which the processes of judgment and reasoning, as they have been applied in various regions of human ingenuity, have led thinkers. I intend ~~then~~ to discuss ~~the~~ certain ~~[illegible]~~ common features which appear in the concepts of different sciences, and which are common to the outcomes of otherwise very different thoughtful inquiries. Philosophers have long been interested in what they have called the Categories, i.e., the most fundamental conceptions of the human reason. I shall try, in these lectures, not to start with any fixed table of Categories, developed, after the manner of Kant, or after the very different manner of Hegel, from

a priori philosophical considerations. On the contrary, I shall first try to consult the experiences ~~of~~ which have accumulated in the course of scientific inquiry, – experiences regarding what types of conception most naturally result from the work of thinkers. These types of conceptions, so far as we shall consider them, will thus first appear to be of fundamental importance, not because any system of philosophy here presupposed makes them ~~[illegible]~~ seem so, but because the experience of thinkers in very various branches of science has resulted in showing that, ~~[illegible]~~

if you judge and infer very extensively concerning any one of a very great variety of types of facts, you come to form and to use these fashions of conception. That the forms of concepts which may thus be empirically shown to be so important, as the outcome of the thinking process, must borrow their importance from something that lies very deep in the nature of thought itself, will of course seem to us (from the very outset) probable. We shall be led, therefore, in our closing lectures, back to the philosophical problem of the Categories. We shall try to see, as we close, whether these important

types of scientific conception are so because of something in the nature of things something which, apart from the interests of our thought, forces upon us these ways of conceiving things; or, on the other hand, whether it is rather due to our nature and interests as thinkers that we find ourselves disposed to view things through these somewhat monotonous conceptual forms. Hence we shall, in the end, raise the question which, at the outset, we ignore, — the question whether there are any fundamental Categories of thought, deducible from an analysis of the very nature of thought itself. Thus, by the way of an empirical examination, we shall approach a philosophical issue, — the issue about the fundamental nature of the thinking process, and about the relation of Thought and Reality. To that issue the concluding discussion of this series will be devoted.

The earlier part of our work however, will be devoted to presenting an account of some doctrines about concepts and (so) will be a very modest and imperfect report of some of the inductions of a new kind of empirical science. This is the science of what I am accustomed to call the Comparative Morphology of Concepts, — a science in which I should like to interest you, and a science which, in the course of

the next generation, may be expected to make great strides. It will be the business of this new science, as it developes, to study the conceptual forms to which the experience of the various special sciences leads thinkers, as they study their different regions of human experience.

You already know of one or two inductions that belong to the scope of this science of the Comparative Morphology of Concepts. You are, for instance, well aware that all men use the concept of Number, — and do so whether they deal with facts of the inorganic or with those of the organic nature; — yes, whether they devote themselves to business, or study theology.

So the applicability of the number-concept to all sorts of distinguishable facts is a commonplace of our childhood; and ~~this~~ obviously this is a truth that has to do with the comparative morphology of the most various results of thinking. In recent times, the much more highly specialized conceptions of ~~the~~ statistical science prove to be applicable to the problems alike of economics and of biology, of insurance and of psychology. Here again (viz. in this wide range of application of statistical concepts), is a fact regarding the comparative morphology of the thinking process as it goes on in various sciences, and also in very numerous applications of science. To mention yet another conceptual form which is of extremely wide application, — you are all aware ~~of~~ what enormous

importance, in very various sciences, belongs to the concept of Rhythm, or rather, as one might better say, the collection of the conceptual forms ~~which~~ that have to do with the description of rhythmical (or periodic) processes. Physical science, with its theories of wave-movement, and psychology, with its studies of the relation of rhythm to consciousness, equally illustrate the significance of this conception. Nor is the conception without important applications to physiology, to sociology, and to the theory of music. ~~[illegible]~~ A recent writer, Bücher was led to study rhythm because of its relation to certain economic phenomena. But to consider the forms and uses of this concept of rhythm, is again to illustrate what I mean by the Comparative Morphology of Concepts. My early lectures will be devoted to illustrations of this sort of study.

II.

Now in ~~all these cases~~ each of the concepts just mentioned, (the case of the ordinary concept of number, in the case of the statistical conceptions, in case of the concept of rhythm, the question naturally arises whether the wide application which any such a concept finds, in the study of very different realms of fact, is due rather to the peculiar nature of things, as they may be supposed to exist and to go on apart from our thinking process, or whether there is something in the inner nature of our own conceptual process which insures to these concepts a certain plasticity, such as makes them very widely applicable to facts, however the world as it is supposed to exist, in itself, apart from our thinking process, may

chance to be constituted. Obviously a complete answer to any such question would require a whole system of philosophy. An empirical study of the actual conceptions present in various sciences shows us that this wide range of application for such concepts as the ones mentioned does exist. So much, to be sure, thus becomes clear, without any philosophy. One needs no finished table of Categories, deduced, as Kant's was, *a priori*, in order to find out that the concept of rhythm applies to music, to the phenomena of light, to the sequence of periods of financial prosperity and adversity, to the habits by which workmen learn to coöperate in certain forms of manual labor, and to many other classes of facts.

But when the question is raised, Why is such a concept so widely applicable? a satisfactory answer is not as easy as might at first appear. (In our closing lectures we propose to study problems of this kind.) Let us glance for a moment at these questions in the form in which they may be raised regarding number, statistics, and rhythm. Numbering is possible in very various branches of thoughtful inquiry. In particular, the whole numbers are very widely applicable. Why? "Because", so common sense is tempted to say, — "because the real external world consists of diverse entities which can be numbered. The discrete structure of the world is a universal fact of experience. The world consists of units and of collections. The whole number-concept expresses a mere recognition of this fact on the part of the thinker." But, on second thought, we see reason to question whether this common sense account is complete. For one thing, the concept of the whole numbers (as this concept now exists in our minds) bears many

marks of its dependence upon purely logical considerations (such as arise out of the inner interests of thought). Number has properties that cannot be wholly derived from a mere description of observed external phenomena. What some of these properties are, we shall later see. In order to define the series of the whole numbers, you are obliged, as Dedekind and several other writers (on mathematical logic) have recently shown, to take account of no ideas that involve any perception of external physical objects. The whole numbers may indeed be called, in a logical sense, the free creation of the human mind, as Dedekind himself declares them to be. But still further, even in applying the whole number-concept to external things, we are not obliged to wait for the world to furnish us from without with any particular degree or type of discreteness of structure. Our attention fixes, often in highly arbitrary fashion, upon the outer facts that we first distinguish, treat as units or as definite collections of units, and then count. It is not merely that we find units in the world, it is not this alone which makes our number=concept so useful. For constantly, by our attentive fixation, now of this object and now of that, we make units for ourselves. The human mind, then, (at the very least) coöperates with the real world in making our own number=concept so widely applicable. Exactly in how far the success of our numerical conceptions in our dealing with (various types of) things is due to us, and in how far it is

forced upon us by some external nature of things,— this is a matter which, despite the familiarity of the number=concept, is not easy to answer. Such questions then, lead us back from the empirical study of the wide range of usefulness of numbers, to the problems of the philosophy of number.

¶ In case of the more specialized concepts of the statistical sciences, it is plain to anybody that, while statistical science deals with external facts which we (are accustomed to) distinguish from all the products of our own conceptual processes, still the choice, the arrangement, and the use of these facts, in the case of statistical investigations, is very highly artificial. In the outer world, individual men live and die. In mortality tables, certain results of statistical study are reported in forms which are obviously very largely due to our own ways of conceiving things. What we ~~want~~ ~~to~~ should like to discover (about the world of outer facts), if we could, ~~to~~ is, What caused the death of ~~each~~ any man (in question) who has died, and also, When each man now living is going to die? A mortality-table is powerless to throw direct light upon any such questions. Instead, it informs us regarding ~~the~~ certain relations between age, or other such conditions (on the one hand) and an abstract numerical object ~~thing~~ called the "death-rate", or another abstraction called the "expectation of life", on the other hand. The information thus given may be of great importance for the purposes of insurance, or for consider-

tions relating to the public hygiene. But it is evident that the skill of the thinker has much to do with the (success of the) forms of conception here used, and that our experience of outer facts forces no statistical tables upon us, unless we have first learned to take a great interest in statistical concepts for their own sake.

If, however, the question arises why the statistical conceptions have so wide a range of usefulness, one answer that readily suggests itself to us is that we use statistics when we are endeavoring to learn about certain general laws and characters of objects which experiences ~~gives~~ furnishes to us in considerable groups, but which we are unable so to classify that we can get make the sorts general assertions that we should like to make about what holds true of every object in a given class. In such cases we have to treat facts in the lump, (numbers of collections and so classes). Thus when we are once sure that the angles at the base of any isosceles triangle are equal, nobody cares to study statistical tables for the sake of finding how often measurements of the angles of isosceles triangles have apparently verified the theorem. But if the eyes of men vary as to the degree and sort of astigmatism of their lenses, while

no assertion is possible to the effect that every man of a given recognizable type, — say every red-haired man, or every man five feet and ten inches tall, must be subject to a given type of astigmatism, — then statistics relating to this and to other variations of men's eyes become interesting. They do not give us rigid laws; but they help us to a study of variations. If a law were known that connected all every definable sorts of business enterprises, in general ways, with the occurrence or non-occurrence of bankruptcy in a given year, statistical tables of the commercial failures of that year would have interest only as helping to what sorts of commercial ventures had actually been made (within the time in question). As it is, the statistics of failures are of interest as throwing some light upon the general state of the prosperity of the country (during a given period). If every man could look into the book of fate, and learn just when he was to die, mortality tables would lose nearly all of their present interest.

Statistical conceptions are therefore most useful when our knowledge of individual facts, and of general laws such as predetermine facts, is in a certain stage of (imperfection and of) progress. The range of the application of statistical concepts is thus determined, not so much by the nature of external things,

as by a certain (often transient) state or stage in (our imperfect) knowledge regarding the nature of things. Hence, if we ask (the somewhat philosophical question as to) why our statistical conceptions are so widely applicable in science, the answer has to be stated rather in terms of the condition and the needs of our own thought about things, as it is at any time, than in terms of the structure of any world that exists apart from our thinking process. We can use statistics so widely, because we are so frequently ignorant of what would release us from the need of statistics. If we pass to the third of the types of concepts just mentioned, we meet, however, with a more difficult problem regarding (the philosophical reasons for) the range of usefulness of our concepts of rhythmic processes. The concept of Rhythm is very widely applicable in science, then at first sight it does indeed seem proper to say that the reason must lie mainly in the nature of things, and not in the nature of our thinking process. Light and sound depend upon periodic physical processes. Generation and decay in the organic world, the weather, music, poetry, the social life, all contain numerous rhythms. This seems to be a law, not of thought, but of the real world. We learn to conceive things in terms of rhythmic processes, because this world of ours is, apart from our thinking process, a largely rhythmic world. And this result, if true, would seem to possess considerable philosophical interest. Such, I say, seems to be, at first sight, the answer to our question as to why the concept of

rhythm is so widely applicable. Herbert Spencer, at the outset of his Synthetic Philosophy, in his First Principles, consequently undertook to define a general law of phenomena which he called the Rhythm of Motion. This law he regarded as holding true of all but a very few exceptional motions which take place in the world. He considered such rhythmic phenomena as those which occur in the economic and in the psychological worlds to be secondary results of the general law of the rhythm of motion, when that law is taken in connection with the principle of the correlation of the various types of forces existing in nature. But now a critical reader of Herbert Spencer's

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chapter on the rhythm of motion, although fascinated, at first, by the wealth of the illustrations, begins, before he has finished to doubt whether the result is of much value for philosophy. (For he) comes to question whether the concept of rhythm is not so widely and so loosely generalized by Spencer, before the chapter is done, as to lose all definite significance, in so far as it pretends to be a portrayal of the nature of things. (For instances of looking for examples of rhythmic processes,) If we consider the weather, we observe how it does not always rain, but sometimes rains and sometimes does not; and that, when it rains, "always there are fits of harder and gentler rain". Spencer uses such facts amongst his illustrations. Now this is true indeed, but in conforming itself to (the whole wealth of) such variations of the weather, the concept of rhythm, as Spencer employs it, seems to become equivalent to little more than the statement that the weather

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pretty constantly changes. Of course the seasons have a more precisely rhythmic character. But the special weather changes are rhythmic only in a much vaguer sense. If you classify weather, with reference to precipitation, into wet weather and dry weather, it is plainly impossible that (in any universe where there is both wet and dry weather) anything should happen, except either a persistence of one type altogether (at a given place) (a state of things which is quite opposed to the hypothesis that in this respect also the weather is everywhere changeable), or else a passing from one of the two sorts of weather to the other, and back again. And if it rains, in case it does not always continue to rain in a perfectly uniform way, the downfall of rain can only vary by either increasing or decreasing; and since the range of variation here is limited, so long as the rain cannot turn into an indefinitely heavy flood, there is no way open to the rain, in case it persists for sometime, yet never long persists uniformly, except the way of coming in fits of harder and gentler rain. Now is this the law of rhythm as applied to the weather? When we pass from the more exact rhythms of the seasons to the special weather changes within [illegible], If so, our [illegible] appears to be a mere commonplace of the logic of any changing process such as often attains the limits of the range to which its variation is limited, rather than to be an important law of nature. The special changes of the weather appear to be rhythmic only in case the concept of rhythm loses nearly all its definiteness. And many other instances of the rhythm of motion, as Spencer gives them, appear to involve little more than the thesis that all natural movements and processes are of a more or less wavering type. This however is equivalent to saying merely that the movements in nature do not take place in straight lines, and that nature's processes do not follow any single direction. If you draw on a plane any visible line that is not straight, and study its relation to arbitrarily chosen coördinate

axes, it can, at any point, only be directed either towards or away from or parallel to any one of the coördinate axes; if it is an ordinary curve, given portions of its course must be either convex or concave towards this axis; and if its course is complicated and widely varying, this variation, considered with reference to the coördinate axis in question, must needs show an alternation of increasing and decreasing distance, of convexity and concavity; and this alternation you may call rhythmic if you will. But by such means you do not discover an important and novel law of external nature. You discover at most, merely, the limited possibilities of variation which are open to you under the conditions

of space and time. What has been said of
the rain, which, unless it is absolutely
steady, must either increase or decrease,
and which, because it often reaches the
maximum of its range of variation at a given place,
[illegible], must alternate between increase
and decrease if it long persists in varying,
– this, mutatis mutandis, holds of any other physical process which involves (the constant change) of
an intensive or extensive quantity of any
kind, and which is subject to a type
of change such as forbids indefinite per-
sistence in either the increase or the decrease (of the quantity).
Of course not all types of change are subject to this law.
A large hot body, left permanently in a
cold conducting medium, to cool by loss of heat through
its surface, by conduction, would (continue to cool for an indefinite time, and would) not vary rhythmically
in its temperature; because its ideal permanent state
of thermal equilibrium with the environment

would be an indefinitely remote limit,
which it would endlessly approach at a slower and
slower rate as its own temperature came to
be nearer that of its environment. But
(on the other hand) if a body (like an engine boiler, sometimes heated and sometimes not) is subject to physical conditions
which constantly change its temperature,
between certain limits which it actually
reaches, yet never exceeds, then of course
its temperature must oscillate up and
down, merely because there is no other possibility open to it.
Or again, since the United States Treasury
can only either take in or give out money, and
does not increase or decrease beyond certain limits its
stock and must constantly do a varying business, the stock of
money in its vaults must needs
sometimes increase and sometimes
decrease. To generalize a law of the universal
rhythmic motion from such instances tells us
nothing of importance about the real world.

Yet, on the other hand, as the experience of the sciences shows us, the actual value of a much more exact conception of rhythm than Spencer's defines, — the value, I say, of this conception, in enabling us to understand and to describe complex phenomena, is very much greater than such considerations as the ones just urged would lead us to expect. The (most useful form of the) concept of rhythm is a very precise and mathematical form — the concept of what is called a harmonic movement. This concept is at once extremely exact, and marvelously plastic. It is exact, because harmonic movements are defined by means of certain functions based upon the simple circular functions of trigonometry. It is plastic, because purely mathematical considerations show that, by properly combining sets of suitably chosen functions dependent upon circular functions, you can

produce conceptual structures which will approximate, as nearly as you please, to the vicissitudes of any chosen physical process of finite length whose character is such as to permit a curve, to be drawn which will represent any definite aspect of what takes place in the course of this process. Draw at random any line of limited length on paper. Then to say that, because that line is broken or is crooked, it has a rhythmic structure, seems, and is, at first sight, quite unenlightening. But a certain mathematical theorem, — Fourier's theorem, — shows that a series can be constructed whose terms depend, for their values, upon trigonometric functions, and whose nature is therefore such that it can express the result of superposing one set of (definitely rhythmic, that is, of wave-like harmonic) movements upon another, while this series can be so built up that the resultant movement to which it corresponds will describe a curve such as will approximate, as nearly as you wish, to the given arbitrary line with which you started. In the same way, let a physical process involve any changes that you please in a limited number of measurable variable quantities. Consider any given portion of this process which possesses finite duration. It will be possible, by applying Fourier's Theorem, to construct a definite set of rhythmic variations of the quantities concerned such that, if all these rhythms are supposed to be superposed,

their resultant will be a conceived process which will approximate, as nearly as one chooses, to the actual physical process in question.

The result of these considerations leads to a law somewhat different from Spencer's generalization. It is the law that the perfectly exact concept of rhythm (which is permitted by the definition of harmonic movements), has such plasticity as to enable us, by combining in the proper way a set of suitably chosen conceived rhythms, to obtain a description which will represent, with indefinitely close approximation, the course of any arbitrarily given movement, of finite duration, or of any limited section of a physical process that can be described in terms of movements at all.

Thus we see that, on the one hand, the effort to start with the given facts, and to attemp[t] without further exact definition, to descr[ibe] the changes of natural phenomena in ter[ms] of our popular idea of rhythm, seems, when taken alone, to lead us, more and more as we proceed, to an increasing vagueness in our conception of what rhythm is. For we then start with an inexac[t] but still fairly clear empirical notion, derived from observing water waves, pendu-lums, and other swinging and rotating mech-anisms of various sorts, (from observing the succession of day and night (and of the seasons)), as well as from an experience of our own activities in singing, dancing, walking, and the like. We are then gradually led to extend this notion of periodic movements so as to make it include the more or less periodic changes which occur in the financial

world, in the weather, and finally in movements and processes of all kinds. We hereupon observe that in our world of change, everything alters in complex ways, and that all complex movements (and changes) are more or less vaguely waver-ing, so as to involve alternations of increase and decrease, of approach and recession, of advance and retreat. To all these things we try to apply our original concept of rhythm, until, by the extension, it loses that character of implying fairly definite periodicity (in nearly equal intervals of time) which it had when we began. At last we reach with Spencer a law of the universal Rhythm of Motion, ~~which~~ (But when approached in this way, our law) amounts to little more than the assertion that, in this complicated world, where nothing moves in perfectly straight lines, all things more or less waggle. Now such a law is not enlightening.

But, on the other hand, the more exact applications of the concept of rhythm in science are due to a wholly different logical process. Of this process in its highest application, Fourier's theorem gives the definite statement. The success of the concept of rhythm is thus at all events very largely due to the remarkable union of exactness with plasticity of which that concept itself, like the concept of number, is capable. We are not obliged to say that, by a mere process of abstraction, we can first render our popular notion of rhythm so vague as to enable us to call anything that happens to occur more or less rhythmic. But we are able to show that, without

sacrificing any of the logical exactness of our conception of a harmonic function, we are able to define a combination of harmonic functions, of various periods, — a combination so complex, and yet so definite, that the result of this combination serves to express any limited series of physical changes whose phenomena are subject to exact quantitative measurement, and are so correspondent in their characters to curves drawn on paper. In consequence, the success of the concept of rhythm, for the purposes of descriptive science, appears to be due at least quite as much to our own methods of forming and of combining our conceptions, as to the rhythmic character of the processes of any world external to the thinker. Any finite series of natural changes,

in case the phenomena are all expres
in terms of a limited number of varia
and measurable quantities, can be
described as equivalent to a superpositi
of exactly definable processes
possessing the type of harmonic rhyth
But that this, each, is true, is due to our own logical power
combine our concepts in a certain way
On the other hand, it is of course also tru
that the more or less exactly rhythmic
character of countless less precisely describable
natural processes, such as the beating
of our hearts, the movements of our limbs,
the play of our voluntary attention, and
the vicissitudes of the commercial world,

depends upon facts which cannot be thus
explained, as deductions in our logic. These facts are not primarily logical
but natural facts. Our final result, therefore, is
that the success of the rhythmical
concepts in their application to experi-
ence is due to a combination of logical
and of extralogical factors, a combination
that could only be rightly estimated in
case we first better understood the
logical processes involved. A comprehensive philosophy of rhythm is therefore a task dependent upon numerous considerations, some of which are logical, while some are empirical.

¶ I have studied in this way
some of the questions suggested by these three typical
cases of the number-concept, the statistical
concepts, and the concept of rhythm,
in order to show you that the problems
of the science which I have called the morphology of concepts are

problems that possess no little philosophical interest. Comparing various sciences, (as you now have seen) we find the same concepts present and useful over a very wide range of investigations, belonging to different branches of study. Such a comparison at once suggests the problem: <u>Why</u> are these concepts so widely useful? Does the explanation lie wholly in the nature of things, as this nature is supposed to exist for itself, apart from our thinking processes? Is the real world, viewed as something totally sundered from the descriptive work of the thinker, a realm where the laws of number reign supreme? Is it a world that is in itself a sort of

treasure-house of statistics? Is it, apart from all our logical interpretations, a region full of rhythmic processes?

We have seen that no one of these questions admits of a perfectly simple answer. Our thought invents numbers; our attention (constantly afresh) gives to our experience that discrete character which makes it possible for us to count groups of units. On the other hand, what we call the real world certainly coöperates in bringing this very result to pass, and furnishes to us much that stimulates us to distinguish units, and to count collections. As for statistical science, — on the one hand it interests us largely by reason of our ignorance of those general laws which, if we knew them,

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would render our statistical collections and arrangements of facts uninteresting. But still, on the other hand, our external experience doubtless, nevertheless, brings to our notice, the classes of facts which make statistical work possible. And finally as regards rhythms, it is perfectly that experience presents them, in a more or less inexact form, to our observation. It is also true that much of the success of our application of the more exact concept of rhythm is due to that logical development of the concept which Fourier's theorem has expressed, and which ensures to the concept a service whose range is as wide as the range of those definable physical

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occurrences which are describable in terms of precise measurements of varying quantities. Here, again, the nature of things, and the nature of thought, come into a very complex interaction and union. And so, in these three typical cases, we have seen how a study of the Morphology of concepts may be needed to define for us philosophical problems which concern very deeply the relations of Thought and Reality. When we approach such problems by this road, they get a concreteness and fairness of definition such as it is hard to attain in any other way.

I propose then to deal with problems of the type just illustrated. I shall first enumerate the concepts which I propose to study. I shall then set forth enough of the elementary theory of the processes of judgment and reasoning to enable us to see by what sorts of logical processes these concepts are reached. I shall then ~~[illegible]~~ try to show you what light the actual success of such concepts in the work of science seems to throw upon the true ~~[illegible]~~ relations which hold between Thought and Reality.

III.

It remains in concluding this lecture to point out, in a summary way, the types of concepts to which I am to ask your attention.

The examples of concepts so far chosen viz., Number, the Statistical concepts,

and Rhythm, are merely special examples of certain general types of concepts which the sciences find very widely useful. I next want to define these more general types of concepts in a preliminary fashion, so as to map out our field of further study.

If we ask ourselves, upon the basis of our general knowledge, what are the types of conceptions which all the sciences most constantly use, ~~the most~~ a very familiar answer would ~~be~~ run much as follows:– All science, for the first, classifies facts; hence the concept of a Class is one of the most general of scientific concepts. All science seeks for the causes of things; hence the concept of Cause is the second most general one of the scientific concepts. All science searches for the laws to which (the connections of cause and effect) as well as the relations of coexistence in the world, are subject. Hence the concept of Law is the third of these universally applicable conceptions.

This somewhat popular account has its value; but it also has its notorious vagueness. Of course we all classify; but there are very different sorts of classes. You can, if you choose, put any objects whatever together into one class, — all the objects for instance that attract your attention during one of your walks; or all

the things that are to be found in the shop of a given dealer in old junk. On the other hand, nature furnishes to you the facts that form the motives for what is sometimes called the more natural classification of certain objects, as in the organic sciences. A natural class is supposed to be no mere random collection of objects. But the question What, if anything, is meant by a natural class, or by a natural classification? is a problem that is known to be one of much (logical) difficulty. Again, some classes of objects, such as the ones just mentioned, are merely ideal collections of things, whose members are not arranged in any particular way. But, on the other hand, many classes known to science are ordered collections of objects, so that the order seems to be an essential part of the very nature (of the objects). Thus, the whole numbers form a class of objects. But this class constitutes what is called a series. It has a first member, a second member, and so on; and every whole number has a determinate place in that series, coming either ~~before or after any~~ earlier or later in the series than does any other whole numbers with which you may choose to compare it. The points on a given line, also constitute a class of objects. And this class also has an order, much more complicated than is the order of the whole numbers, but still a very precise

order. Historical events may be classified. Thus the events of the French Revolution ~~[illegible]~~ form a class. But this class has a chronological order, which determines (to a great degree) the way in which the historian recounts each fact. There are many classes known to science whose members ~~[illegible]~~ conform to still more complex types of order; for example, the points in space (form a class of objects, but also) constitute a tridimensional order. Classes then are sometimes mere random collections, (sometimes what are called natural classes,) and sometimes systems of various degrees and types of orderly complexity.

You gain little, therefore, by knowing that all science classifies objects, unless you have some con=

ception of the numerous types of classes which exist, and of their relation to one another.

As for the second of the three universal types of concepts mentioned, namely the concept of Cause, the vagueness of our idea of what we mean by a cause is a commonplace of Logic. No highly abstract term is more familiar; few general terms have a less definite meaning. We shall hereafter see that no logical treatment of causation in general is possible, just because the term has no precise definition. In order to deal in any exact fashion with those concepts which are usually called

concepts of cause and effect, we shall have to use entirely different names from the ones now most in popular favor.

And thirdly, as to the concept of Law, this is indeed, in most scientific usage, a much more definite and precise term than cause; but it is still an unfortunately vague term. The mere fact that the term has its legal as well as its other technical usage, is sufficient to show how dangerous is the conflict of associations which its use invokes. The well-known question as to whether a law does not imply the existence of a law-giver, is an instance of the artificial and needless problems to which this ambiguity has given rise in the popular controversy. We shall find it necessary ~~needful~~, in our discussion of scientific concepts, to make frequent use of other terms than the term law, although of course we shall not avoid that now inevitable term when occasion demands, and when our meaning is clear. We shall try, however, to make a little plainer ~~clearer~~ the place that the concept of law, rightly understood, occupies among other widely used scientific concepts.

The three terms Class, Cause, and Law, are therefore, for our purpose, very imperfect expressions for the types of scientific concepts which we shall

have to consider in our general sketch of the Morphology of Concepts. Instead of confining myself to them, I shall begin to outline our field in the following way.

The concepts of which, in the following we wish to take some account, are, first, certain very fundamental and elementary concepts upon which, as we shall see, all else in science depends; and secondly, certain derived concepts, of a more complex structure, which will especially interest us in the course of these few lectures. Concerning the first sort of concepts, the more elementary ones, I shall be as brief as possible.

The others, the more complex concepts, at the next time will interest us more. But, as I close this lecture, I have time only to [illegible] the most fundamental simple and elementary [illegible] of the concepts of [illegible] of the first or elementary fundamental concepts, there is this to be said here, by way of mere preliminary:- All thinking depends upon fixing the attention now upon this object, now upon that. That upon which you fix your attention becomes in your mind, at least for the moment, a relatively isolated fact, something abstracted or held apart from all other objects. Such an object, which you view as this thing, or as this quality, aspect, portion, or feature of something, I venture to call an Element. By an element I mean such an object as this

point on a line, this digit, this star, this man, (this tree,) this item (in the newspaper), this planet, this stellar system, this color, this tone, (this flash of moonlight on a wave) this abstraction, this logarithm, this sine or cosine, this [illegible], or any other this, in heaven or earth, which your attention may chance to isolate, for the purposes of thinking about it. An element is whatever you may count as one, or name by a single term, or hold for itself before your mind. Now whenever you think, you think of certain elements, however transiently you may fix your attention upon them, and however ill they may correspond to any profound truth about the nature of things. The essence of the concept of an element is that an element is selected by your attention, and is dwelt upon, for the moment, as a single fact.

Now I call any *this* upon which your attention may be fixed an *element*, because, as a fact, you never thus dwell upon elements without at once going on to view them along with other elements, as entering into collections, classes, series, or other more or less systematic *groups* of elements. Your fixing of attention upon a singular element is always only a beginning of a further thinking process, whereby your new element gets its place in some sort of system. Hence the element (as you first observe it) exists for you only as a starting point for further inquiry. When I dwell upon *this* object, whatever it is, I next proceed to group it with other objects. I classify it, determine its place in a series, tell what characters it possesses, tell where and when it is, try to explain it, and, in general, look for its place in the world.

What my attention first finds, is then an element. What my thought does with elements is to place them in what we may now venture to call Complexes. I here use the term Complex for any collection, class, arrangement, system, or order of facts, or whatever, be this complex of whatever grade, high or low, chance or rational, transient or of permanent significance. I prefer the term Complex to the other term, Class, for reasons which will appear more clearly at the next time.

All science then, uses conceptions of Elements, and Complexes. Complexes are collections or groups of objects, either taken as without order, or ordered in some way. Now in our following lectures

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we shall be concerned with studying certain very widely useful types of complexes. In particular we shall deal with complexes which I shall call by the following names:-

1. Simple Series.
2. Domains.
3. Transformations
4. Levels.

I shall try to show you what a wide range of scientific concepts can be reduced to these four types. I shall also try to show you how these four types are related to one another, and to the interest of our thinking.

Apparently Incomplete

Lecture II. General Survey of the Concepts useful in various Sciences.

In the former lecture we sketched the general problems to which this course is to be devoted. ~~We take~~ The starting point of our whole inquiry is furnished by the ~~fact~~ observation that widely sundered branches of scientific inquiry, although they ~~seem~~ may deal with (what at first seem to be) extremely various sorts of facts, still find, in many cases, the same types of concepts useful for the purpose of dealing with these different