Josiah Royce

The Richmond Lectures of 1904

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The records of that university show that Professor Royce gave three lectures as follows:

I

Thursday, November 3

"The Orderly Arrangement of Facts and Ideas: 'Series' and 'Levels'"

II

Friday, November 4

"Transformations and Their Laws"

III

Saturday, November 5

"The World and the Will"

J. Harry Cotton

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1904

The Richmond Lectures
on
"Some Fundamental Conceptions of Science"

Lecture I: The Orderly Arrangements of Facts and Ideas: Series and Levels.

I propose, in the following lectures, to point out the philosophical bearing of a few ideas with which, in a sense, you are all more or less familiar. I have called these ideas, in my general title: -- "Some Fundamental Conceptions of Science". In this lecture I shall try to tell you what these ideas are, and why they are [2] fundamental, and why I have ventured to trouble you with the mention and examination of them. Since there is a good deal to say about these ideas, you will forgive me if I spend little time upon introductory speeches, but plunge at once into the heart of my undertaking. One word only, as I begin, regarding my personal attitude towards this audience and this undertaking.

You have very kindly invited me to this institution, and to this community, to address you upon [3] some topics connected with philosophy. I delight at this opportunity to become more closely acquainted with you, and to further the academic and personal relations between your community and ours.

My lectures, as I fear, if judged by their titles, and perhaps too if judged by their contents, will seem to you to deal with very dry and remote topics. I shall be sorry if this impression proves in the end to be the most prominent one in your minds; but as a fact I want, so far as I can, in the brief time, to show you that precisely these problems, dry and remote though they may seem, have a very real [4] relation to the practical lives of all of us. If I fail in this effort to apply philosophy to life, it will not be from good will. Nor will it be because philosophy itself, if rightly interpreted, is uninspiring. In any case, I beg you to be patient if especially the earlier portions of my discussion seem to you to be technical, and sometimes hopelessly distant from practical interests. Before I am done with the whole task, I intend to reach less arid topics, and more obviously vital considerations. In the present lecture then, and in the next one, shall be at some point concerned with decidedly abstract matters; and some of these will seem to be decidedly [5] forbidding. If you can bear with me while I speak of these matters, what will follow, in the third lecture, will be, I hope, much more interesting.

I.

One of the principle problems of the thought of the present time is this: How much can our natural sciences tell us about the true nature of the world that we live in? To this question you will find a great variety of answers given. If, in the first place, you ask the special students of any one science, such as physics or zöology, what that science teaches us about [6] the universe, you will generally get, in substance, the answer that physics tells us about certain properties of matter, and does not attempt to tell us any more about the true nature of things than physical experience itself enables us to verify; while zöology investigates animal life, and does so by the aid of observation and experiment, and tells us precisely as much about the nature of things as its own range of observation and experiment reveals, and no more. Were this the only sort of answer that we ever got to the question about what it is which the special sciences of nature reveal to us concerning the [7] universe, we should of course by no means lose our keen interest in the existence and in the progress of the natural sciences. We should honor physics for the vast control over natural phenomena that it gives those who know it, who advance and who apply it, and zöology for the enormous mass of knowledge of the processes of animal life which it is gathering; but we should not feel the concern that some of us feel in the true relations between science and philosophy, or in the bearing of science upon religion and upon men's general attitude towards the problems of life as a fact, however, there are some students of the special sciences, and many philosophers and theologians, who do not rest satisfied with this first answer. Some of these students give us a second answer to this question, -- an answer which you all have heard. They tell us [8] namely, that the lesson of science is wider and deeper than the lesson which you could learn by merely mastering the details of a large number of special sciences, were such a mastery possible to you. This lesson of science is, they say, that the comparative, i.e., by the distinctly philosophical study of the combined work of many sciences something decidedly definite has been made out concerning what the whole nature of things is. Science, they say, -- not any one science alone, -- but a science as an organized body of research, has taught us that the whole universe is subject to certain types of law, -- mechanical law (as say some such students); -- evolutionary law (so say others); while some men (for instance such as the late Herbert Spencer), attempt what they [9] regard as an unification of science and try to show us how some vast process of evolution, which includes in its sweep physical, chemical, vital, mental, and social processes, results from the very mechanism of things, and even the entire knowable world is a treasure house of examples of a single type of natural processes. Some of these unifiers of science prefer materialistic formulas; some of them attempt to reconcile science and theology; some of them try to interpret the entire world as an expression of a mental process, -- that is, of a process similar to what goes on in our ours [sic] [10] minds, when we think and choose. On the other hand, many of these general students of the supposed lessons of science lay stress upon the limitations of our power to know the nature of things,-- limitations which, as they say, the very study of science emphasizes. Spencer himself, as you know, drew such a lesson, when in one part of his system, he defined, so far as was possible, the Unknowable, and showed, as he held, how hopeless it is for man to try to penetrate the mystery of the true nature of the universe, as it exists apart from our experience. Many others, using still other methods and arguments besides those of Spencer, tell you that the very [11] successes of science, showing us, as they do, that all our actual knowledge of things depends upon observation and experiment, prove that we can never get, in our study of nature, beyond precisely what observations and experiment verify; so that any philosophical effort to define the ultimate truth of things must necessarily fail. According to such views, the general lesson of scientific study is that you never can hope to find out, for instance, whether God exists, or whether the soul is immortal, or whether the universe is simply a complex of material atoms, subject to the laws of motion, unless you can [12] submit all these problems (about god and immortality and materialism) to the test of observation and experiment. If the telescope or the laboratory could, by searching, find our god as something observable, or can submit the hypothesis of his existence to an absolutely crucial empirical test, than the problem of God's existence would become a scientific problem, and some special science could deal with it. Otherwise, say such investigators, speculation is vain: god's existence remains unknowable, and a similar result follows in case of the other problems mentioned.

Thus then some of our teachers hope to get from a comparative study of the results of various sciences a genuinely unified and enlightening view of the real nature [13] of things. Some, on the other hand, emphasize the fact that science, by virtue of the very method which has led to its successes, teaches us the lesson that, since observation and experiment, which always lead to special results only, are our sure guide, no enlightening view of the whole nature of things, -- no philosophy, -- can ever be hoped for as a natural result. Some, like Spencer, sharply distinguish the knowable from the unknowable aspect of things, and attempt to verify the one, but forbid us to enquire with the other. And thus, between the frequent glorification of the light that science brings us, and the insistence upon the modesty that science teaches us to practice and the deep ignorance which science [finds to...beyond, and beneath all our researches, (between those ....),][1] we students of philosophy often feel baffled, and so the question: What [14] general lessons does the study of science teach us concerning the constitution of things remains, for most of our minds, unanswered.

As I have pointed out, the special investigators in the various sciences are but seldom responsible for these perplexities. They soberly attack particular problems, and deal with these as they can. If we who philosophize persist in asking more general questions, that is not their fault. On the other hand, indeed, I cannot think that we are wrong in bearing in mind the pressing necessity of these more general problems about the nature of things, or in asking whether [15] either the methods or the results of the special sciences have any definite bearing upon universal problems. The universe we have always with us. You cannot live without behaving as if you had solved the problems of philosophy in some very profound and important sense. For instance, to act dutifully (if you have any reason for doing so), implies a trust in the essential reality of the right. To feel confidence in life implies a faith that this world has an ideal value. Even to undertake the study of special science involves a certain trust in the reasonableness of life of things whose far reaching importance [16] many skillful special investigators fail to notice. The assurance that reason rules the universe is one with which man does not lightly, or painlessly, part. If he parts with it,-- if he even rationally doubts it, -- he even then does so upon grounds (if they have any sense at all) which are essentially philosophical grounds. And so the right to philosophize,-- to attack the world-problem in a serious way, is one of the essential rights of a civilized man. You may not exercise the right; but you cannot lose it, except by your own fault.

I insist then,-- we have indeed a right to ask what the general lessons of science are, and in these lectures I propose to make some beginning in such [17] an inquiry.

II.

Now my method of inquiry, in these discussions, will differ from the method that some of you are accustomed to consider natural. Mr. John Fiske, throughout his career as a public teacher in this country, was accustomed to insist, as are independent followers of Herbert Spencer, upon what Fiske believed to be the positive truth which a comparative study of the results of many special sciences revealed. The doctrine of Evolution which Fiske used so beautifully to expound, was taught by him as such a synthetic result of the discoveries of many [18] sciences. And this doctrine of evolution

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[1] Part of this line is illegible.

as Fiske held, revealed in a very notable respect what the true nature of things is. Here, as you well know, was an effort to read the lesson of science by a synthesis of the generalizations of various sciences. I am not here to criticize, upon the present occasion, the usefulness of that sort of procedure. Only such a procedure is not at present mine, I shall here try to study the general lessons of science in quite another way. I have just reminded you how some people emphasize those limitations of human knowledge which, as they say, the very success of the special sciences bring all the more clearly to our notice. [19] Since observation and experiment, say such teachers, are our only methods of finding out scientific truth, knowledge ceases wherever observation and experiment are no longer our guides. At the present moment I will not as yet criticize this opinion. But the general fashion of studying the lesson of science which this opinion exemplifies will be the fashion that, in another application, I shall here attempt to follow. You can study the results of science, and see whether these bear upon philosophical problems. Or again, you can study the methods, the ways of conceiving things, or the leading ideas, which characterize science in general, or which are notable in various sciences. And you can [20] then ask how far any sort of investigation which uses these methods, which conceives things in that way, or which follows the lead of any such ideas, can hope to throw any light upon the problem of the universe. This second way of studying the lesson of science is the way that I shall here follow. To what final results we shall be led, only a later lecture can show. So then, my question in these lectures will be, what lessons can we derive from a study of the general methods and fundamental conceptions upon which all scientific inquiry depends.

Whatever else our natural sciences are, they are products of human effort and activity. They stand as the embodiment of human interests. They are the result, not indeed of anybody's caprice, but certainly of the efforts of the human will. They [21] are not only theoretical structures. They are the outcome of practical devotion and of personally directed labors. That, in fact, is one reason why they have such close relations to the industrial arts, and such vast influence over the practical activities of these days. I yield to nobody in my admiration for what is called the purely theoretical investigator,-- that is the scientific worker who devotes himself to the truth so eagerly that he hates to hear the question asked: "Of what practical use is this research in which you are engaged?" I love to see a man so confident of the preciousness of mere insight that he throws worldly [22] success behind him, is careless of pecuniary reward, and passes on the treasures of his researches without stint, utterly careless what practical profit others may make out of the consequences of his devotion. Such an investigator,-- a Faraday, a Maxwell, a Darwin, a Gauss, a Newton, may all the more certainly aid in the transforming of the practical procedure, of the industrial art of the whole spirit of a civilization, just because he himself is so devoted to what he takes to be the theoretical pursuits of truth for truth's sake. But what I notice in such a man is that he is, as a fact, engaged, despite the theoretical remoteness of his life, in one of the hardest and most fruitful of laborious human activities,-- the activity of conceiving the relations of [23] things. Thinking is itself a sort of doing. Theory is the building and the testing of ideal structures. And ideal structures are often far harder to plan and to rear than are modern office-buildings. Ideas are often much more refractory materials than are steel beams and girders. The tests that serious scientific theories have to meet are often much severer than are the tests applied to railway bridges. The engineering of a scientific plan is often much more of a task than is the spanning of a continent. In brief, the pursuit of a science is hard work, and it is work done with a human purpose, leading to a humanely interesting goal, and guided by ideals which the universe predetermines only in so far [24] as the universe is man's universe, and only in so far as it permits itself to be interpreted in man's way; so that, in one sense, the lesson of science must indeed be limited to showing us, most of all, not how the nature of things would look if you could leave man and man's mind out of the universe, but how man interprets his own life, and his own experience when he tries to conceive them most soberly, most rationally, and most thoroughly. And so, my first thesis in these lectures will be this: --Science does indeed primarily reveal to us, not what the universe is apart from man, *but how man interprets his own experience*. Thus his interpretation is due to his own active efforts. If he had other ideals, if he worked otherwise, if his interests had another trend, [25] if his senses were awakened through other channels, if his will were the will of some angel or of some demon, and not a man's will,-- then his science would use other conceptions, his universe would get some other appearance, his laws of nature would be defined in other fashions, his opinions would possess some other constitution. In order to read aright then, the lesson of science, you must first see to what end and in what way the conceptions of science are built up. You must see by means of what thoughtful deeds man constructs his world-picture, and in what way this structure of human conception is exemplified now by this science, and now by that. [26] In other words you must recognize that, whatever be the facts of experience, man's conceptions of these facts are the results of the essentially practical ideals which guide even his most theoretical undertakings. In sum, you must learn to estimate the importance of the subjective, of the human, of the personal factor in the life of science.

When I say this, am I once more admitting that the nature of things is *wholly* unknowable, and that we are always dealing, even in the most exact science, *merely* with a human interpretation of things, which bears no trace of possessing any eternal value, or of being the genuine and final truth? No, I am not saying [27] that the nature of things is wholly unknowable. If you wait until my closing lecture you will hear me tell why this is not my meaning. But at present I am insisting that the lesson of science is always a lesson of human life, and cannot be a revelation of what the world would be if our human nature could be left out of it, and if the world could be studied as if man were not alive at all.

Some students of our topic would admit all that I have just said; but would then attempt to illustrate it by laying their whole stress upon the fact that all our ideas of the physical world depend [28] upon our sense organs. They would say, "If we had other senses, if, for instance, we had some new and special electro-magnetic sense, in addition to our present sense of sight, we should perhaps directly feel Herzian waves, so that wireless telegraphy might appeal to us as music now does. "This," they would observe, "does indeed show us how subjective and limited our present impressions of things are." But I am not now laying any chief stress upon the human character and limitations of our sense organs, or upon the subjectively determined quality of our sensory experiences. What I am here emphasizing is rather the extremely human character of our modes of scientific conception, the extremely [29] human character of the fundamental ideals and methods upon which every science depends. In other words, I am laying most stress upon the lesson which Kant first taught us,-- upon the lesson that what Kant called the Categories of our thinking,-- the fundamental modes of conception upon which our sciences depend, are human categories; so that we may not teach as the lesson of our sciences, anything about what the world would be if man, the conceiving thinker, the scientific worker, the organizer of his realm of empirically illustrated ideas, were not an essential factor in the world as he conceives it.

This is what I want to bring to your notice. The philosophical lesson [30] of such a set of considerations I have already faintly outlined; but that we must postpone, in the main, until our final lecture. For the time I want to make you better acquainted with some of these fundamental modes of conception, as the various sciences reveal them. To this task I now turn.

III.

The facts of experience come to us however they happen to come. When we are successful in the work of any of our sciences, we learn to conceive the facts of experience in an orderly way. Science, as you all know, is knowledge [31] reduced to order. The conception of Order then, is the central one amongst the most fundamental conception of science. If we are to reduce facts to order, we must engage in various processes of ideal construction which are usually decidedly complicated and difficult.

As a result of these processes we build up certain systems of ideas to which we find our experience conforming, precisely in so far as our scientific theories prove to be successful devices for predicting and for controling [sic] experience. These systems constitute the various types of order which occur in our sciences. What we mean by order, our own rational ideals determine. And these rational ideals as we have said, are essentially practical ideals. An orderly system of ideals offers to us a certain freedom [32] of movement in which our will takes satisfaction. In getting our ideas into order, and in adjusting these orderly systems of ideas to our experience, we thus enable ourselves to win a freer control over our lives such as industrial art in manifold ways impressively illustrates, while pure science, despite its frequent apparent remoteness from practical concerns, none the less constitutes one form of free and successful and rational living. Understand what order is, therefore, and you will come to understand what science accomplishes. Understand why order is significant, and you will see why science is enlightening, and powerful, and ideal. Understand that any [33] science consists simply in getting some portion of human experience into order and you will begin to see what sort of lesson science can teach us. The assertion that all science is orderly knowledge or knowledge set in order is easy to illustrate. We know many things; but whoever knows a science, is not merely acquainted with many facts, but also possessed of such principles, methods, and mastery, as enable him to hold together these facts as we have now asserted, in an orderly unity of insight. Look about you during a journey, and you shall see countless beautiful features of landscape,-- hills, streams, valleys, cliffs, plains, headlands, mountain peaks. These are facts which you know. These are data for science. But this is not scientific knowledge. Make this journey, however, in company with a geologist, scientifically acquainted with the country in question; and he will begin to reduce these observed facts of human experience to some sort of order for you. These cliffs are, for him, fragments of a certain rock formation, [34] traceable, perhaps, in this or in that direction across the country, and related thus and thus to earlier or later formations. These hills and mountain peaks have their specific relations also to a great mountain system, which, perhaps, extends along the shores of a whole continent. This entire landscape is full of records of what the geologist, using the ideal devices at his disposal, conceives as geological time, capable of conceiving an orderly reading. Geology involves then, in this special case, the power to reduce to a certain order facts of human experience which, for the unscientific tourist appear fascinating, indeed, but disconnected. What this single instance suggests, any science, so far as it is a science, illustrates. Whoever has even touched the edge of any science, knows how the very essence of scientific mastery is the [35] grouping of experienced facts into such systems as the geologist's knowledge has just illustrated. But hereupon let us ask: What is Order? The question as thus stated has indeed no very practical seeming. It seems to refer to a very abstract logical problem. Yet as I must at once point out, it is a question that also meets you at every turn in practical life. And it certainly has very practical bearings. Order, people tell you, is heaven's first law. Order, I should tell you, is in any case the very basis of all human lawfulness of conduct and ideas. Certainly then, this concept of order is not only a scientific concept; but it is a social, a moral, an industrial, an aesthetic, a domestic ideal. And wherever you meet it, its human significance is enormous. Order and civilization go together in social affairs. The moral life is the orderly life. We [36] speak of criminals as disorderly people, and characterize an immoral life as a disordered life. Delirium, and insanity in general, impress us as instances of disorderly mental life. We also speak of any physical diseases as a disorder. Housekeeping uses the concept of order as its most fundamental one. Without order, no domestic life. But order is not only scientifically and practically essential. In many forms it is beautiful. All the effects of art depend upon order of one sort or another. Music is an order of tones; painting, sculpting, and the decorative arts involve order of light and shades, of forms and of colors. Dramatic art has its orderly presentation and sequence of scenes and of acts. Poetry is dependent upon rhythmic order. Order [37] then is everywhere where goodness, beauty, successful living, and power of spirit are to be found. The opposite of order is chaos, or confusion, or distraction, or incoherence, or anarchy, or at best the miscellaneous heaping of things – mere formlessness. All these names for the disorderly suggest to us states of things, or of ideas, which we regard with one or another form of dread or dislike, according as we are thinking of moral of aesthetic [sic], or of scientific interests. Disorder stands for failure of one sort or another. Success, of any reasonable sort, involves order.

Thus then, the conception of Order is not merely a fundamental conception of science, but one of the most fundamental of all human ideas. [38] The postman who orders his letters and the apothecary who arranges on shelves his bottles, the housewife and the artist, the philosopher and the moralist, all equally need and use, in their various ways and worlds, this one idea of Order. The reason why I am especially to emphasize in these lectures the scientific uses of the conception of Order, rather than the other aspects of the concept, is due to my choice to topic [sic]. But in any case order has its distinctly human interest and, as a fact, the concept of order is an essential part of every wholesome and civilized form of consciousness. It is so because we men need order for the sake of our own ideals.

All this is familiar enough. But now once more: What is order? What constitutes order as opposed to disorder? This question is no easy one. Only recently have the philosophical logicians learned how [39] to deal with it. Order is in some respects like what St Augustine declared time to be. Namely, you do not ask me what time is, said Augustine, I know; but if you ask me, then (at least at the outset of the inquiry) I do not know. It is very much so with Order. If some disorderly child, whom we are trying to teach to be orderly, with his playthings, clothes, books, sports, studies,-- should turn upon us suddenly and ask: What is order? – could we answer him? Some of us would find it hard.

Let us, however, make a beginning with this question. And to this end let us turn back, for the time, from the vast complications of the world of art, of social life, of business, and of morals, and try to exemplify what order [40] means in science. Here the various ideas concerned stand out a little more clearly. The instances that we shall have to study will be freer from emotional and other extraneous complications. Our view of what is essential in order to constitute order will be thus rendered more accessible, and in the end more exact.

The simplest instance of a scientifically exact order of objects and of ideas which I can mention to you is furnished by that most familiar of ideal objects, the series of whole numbers, one, two, three, and the rest. I must call [41] your especial attention at once to the fact that, however we may have first come by the idea of number, the number names stand in our minds as the names of a series of possible ideal objects,-- a series of objects which would remain unchanged in character and in properties whatever might come or might go or might change in the physical world. You can view the whole numbers, if you will, as what the mathematicians call a purely ordinal series of ideal objects. So viewed, the numbers form a series which has a first member, but no last member, while every [42] number has a next successor, and every number in the series has its absolutely determinate place, following every one of a perfectly definite class of numbers, and preceding every number which is, as we usually say, greater than this number. It can be shown that by the assumption and exact statement of a very few principles of this type you can give a perfectly sufficient basis for the entire theory of the arithmetic of whole numbers.

The series of the ordinal whole numbers is an enormously important scientific conception. The [43] human reason has grown great upon it. The earliest human science was busied with it. The higher theory of numbers forms today one of the most difficult regions of mathematical inquiry, but one where new discoveries are constantly made. Meanwhile, one may well say that all science would be impossible without the whole numbers and without the fractions which we define upon the basis of the concept of the whole number. Yet, on the other hand, there can be no question that the numbers, as exact science now understands them, are objects whose properties may be determined by taking account of purely ideal considerations. Our mere experience, apart from the rational interests which lead us [44] to count, does not force upon us the existence of numbers, in the sense in which experience forces upon us the existence of pains or of pleasures. We count because we want to count, need to count, find rational and practical satisfaction in counting, and in its result, and gain thereby control over nature, and over our own lives. But apart from the interest which a self-conscious being takes in order, and in its most fundamental instance, viz., numerical order, the existence of numbers is inconceivable. [45]

Now the instance of the number-series is not only our first example of order; it also reveals to us a fundamental property which is always present in those systems which possess any definite order. All such systems, I affirm, are either simple series, or else complexes that are based upon and built up out of series. If the books in a library are in order, that is because the books are arranged in various series, which themselves are formed into series of series; so that by knowing what alcove in a fixed series of alcoves, what row in a certain series of rows in a given alcove, what shelf in a given series of shelves, and what number of a series of volumes on a particular shelves [sic], you are to look for, you [46] can find the book. Precisely so, in just so far as the astronomer's knowledge of the fixed stars is reduced to order, the astronomer deals with a system of facts which consists of series. There is the series of stellar magnitudes; there are statistical series containing the numbers of starts of a given magnitual [sic] which are visible in a certain portion of the heavens. There are such series as the succession of changes of brightness through which a given variable star passes. There are serial arrangements of the spectra of various stars. Meanwhile whatever systems of facts in astronomy, or in any other science, are set in [47] order by means of those countless devices of measurement, which exact science uses, are set in order by being reduced to series, and to complexes of series. For measurable quantities form series of greater and less quantities,-- and that too, whether you are dealing with distances, with masses, with velocities, with pressures, with energies, or with any other measurable objects. The whole value of measurement in science may be logical explained [sic] as reducible to the advantages which result from the exactly definable series which you can get before your mind through the process of measurement.

<u>Lecture II: Instances of Order, Number, Relation</u>[2]

[1]

The simplest and best known instance of order is furnished to us by the familiar series of the whole numbers. In discovering this series the human mind made one of its first steps, and one of its most important steps towards exact science. However we came by the idea of whole numbers,-- and of that subject I have nothing to say on the present occasion – we at present conceive of them as a series of ideal objects, a series marked by certain peculiar characters. The series has a first term, namely number 1. Every term in the series is followed by a term which is called the next member of the series. As for instance, number 2 is followed by the number 3 and so on. The series has no last member. It can be proved, and has been proved in recent discussions of the logic of mathematics, that by increasing the whole number series simply in this way with the additions of a very few formal considerations, one can proceed to deduce all those properties of whole numbers which constitute the vast science of the theory of numbers, including the whole of ordinary arithmetic, so far as that arithmetic deals with whole numbers. To the whole numbers the human mind early learned to add the fractions, although in the earlier stages of mathematics the conception of what a fraction is develops somewhat slowly and in most minds remains comparatively obscure, even after considerable training in the use of numbers. The rational fractions, such as 2/3, 9/10, and the rest form a very complex but very exactly ordered series of ideal objects. If any two fractions are given to you, then by the simple device of reducing them to a common denominator you can tell which of them comes earlier in the series of fractions than the other. The series of [2] fractions is arranged according to the so-called values of these fractions. Just as in the case of the whole numbers one can prove that the science of the theory of numbers may be developed simply by assuming the fixed order of the series of whole numbers, so one may say that the whole arithmetic of fractions depends simply upon the fact that fractions are objects which can be dealt with in certain fixed order. In fact, one can at once say that the whole arithmetic, algebra, analysis,-- that is, the whole of ordinary mathematics apart from geometry, can be developed out of the simple properties of the ordered series of whole numbers and of fractions. This very elementary instance already illustrates both the nature and importance of the conceptions of order. The exactness of mathematical science depends upon the fact that the numbers are objects that stand in precise order.

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[2] This line was added around 1940 in red pencil by Edgar Franklin Wells, then a Ph.D. candidate a Harvard hired by the Department of Philosophy to help organize the Royce papers. It is unclear whether this is, in fact, the Second Lecture, or a continuation of the First.

But mathematical topics appear to the uninitiated extremely abstract and many of you will not be disappointed if I turn to somewhat more concrete instances of order. As I do so, I shall still call attention for the time being to cases where objects are set in order, but where the order has the same character that one may observe in the series of numbers, namely, the objects in question form a simple series of what is sometimes called a linear series, a series consisting of a single row of objects. [Take a glance at any foot rule, or tape measure, or other scale used in the process of measurement. What you see at once upon the scale is an orderly arrangement of dividing points usually accompanied by numbers. Evidently the whole use of the scale depends upon the precise order of these division points and the names that are attached to them form themselves as orderly series upon which all the knowledge that can be acquired through the use of the measure depends. The thermometer that hangs [3] upon your wall derives again all its usefulness from the ordered series of degrees named upon the thermometer scale to pass next to a case of what is not measurable. The houses on one side of the street are in ordered series, numbered in a way which suggests to us in very marked fashion what the value of ordered series may be.][3] But linear ordered series are not confined to scales or to rows of houses. The alphabet forms a single and one-dimensional series of objects arranged in a single row.[4] The various stages of any activity that we can carry out, of any list of names that we can repeat, of any orderly arrangement of the day's work; all such series form instances of the type of order that I now have in mind. Series of this type one might call artificial, since they are more or less due to our own activity as persons who are fond of getting our acts or our memories arranged in serial order, but nature [helps us to arrange in order a large number of series of facts whose serial order, when once we have come to conceive it, becomes of the greatest importance for science as well as in many cases for art. The human voice by its well known power to run up and down the scale furnishes to us a certain natural series of tones. These we can recognize in musical instruments, and to a certain extent in the tones of nature, and upon our power to recognize this tone series depends a great deal of our skill in knowing and understanding facts with regard to those tones. But the sounds that we hear are arranged not merely according to the musical scale, they vary also in intensity, as loud sounds or faint sounds, and any given sound can vary from a greater to a less intensity by gradations which themselves constitute a simple series of the kind that we are here considering.][5] What interests us at present is the fact that not merely that such a series exists or that it has in many ways interest for our feelings; we are especially concerned with the fact that such series are of

[3] Marginal note next to bracketed section reads, "Read."
[4] Marginal note reads, "Om." next to this line.
[5] Marginal note beside bracketed area reads, "Read."

importance [4] to our intellect. They enable us to conceive as we say the relations of tones. The dying away of a sound indicates to us something that may have importance in telling us that the object which makes the sound is moving away from us. The increase in the intensity of a sound may indicate the approach of any object or again it may indicate the increase of some physical process which is causing the sound. In this way, you see that a capacity to conceive the facts of nature or of our own minds in a certain serial order is the basis for a kind of scientific knowledge which if less exact than our arithmetical knowledge, still depends for its reasonableness upon similar conditions. [We may here notice in passing that a part of the world of our experiences of sound is not capable of an easy arrangement in any orderly way; or again, that many sounds are not present to us by nature so that we can easily conceive of them in orderly series. The sounds of the kind that we call noises vary, to be sure, in loudness. They vary also more or less in pitch, but they vary in many other ways that we cannot so easily arrange in series. Noises enable us to recognize the various things from which they come, but just because the noises cannot easily be arranged in simple series as the pitches, or the degrees of loudness, our scientific knowledge of the nature of the variety, of the classification, and of the causes of noise, either remains very much more confused and obscure than our knowledge of musical tones, or if we get such knowledge we meet with many obstacles on the way and where the obstacles are surmounted they are surmounted by means of devices which reduce again the noises to some sort of serial order.][6]

Not only the tones, but the impressions of the sense of sight are capable of serial arrangement. These again differ in degrees [5][7] of brightness. They differ also in color. Our color sensations are of such great variety that the unaided consciousness does not easily discover the precise way in which colors are related to one another, and our popular appreciation of color remains unscientific. Our scientific knowledge of color depends in part upon the fact that by certain well known physical experiments we can form certain series of colors, namely the colors of the spectrum. Upon the basis of the discovery of this series rests very much our further scientific acquaintance with the nature of color. In the spectrum we see our colors arranged in order, and having once seen this order we are capable of making further investigations into the relations amongst various color experiences that do not at first appear in the spectrum.

If we turn from the world of the colors to the world of the odors we get a striking negative proof of the importance of order for all our scientific insight. Of the odors we have comparatively speaking very poor ideas, despite the wealth of experience that the

[6] Marginal note beside bracketed passage reads, “Omit.” Note at the top of the page reads, “Read the whole page except for the marked passage.”

[7] Note at the top of the page reads, “Read until marl.”

sense of smell gives us and despite the very great interest that many of us take in the odor of perfumes, of flowers, or of foods; but the odors in the present stage of science defy our efforts to arrange them in orderly series. They are unlike the numbers, unlike the experience of musical pitch, unlike the colors of the spectrum. They are various, but they remain for our experience disordered and for that very reason our scientific knowledge of them is small.

[In another realm of our experience order again meets us, but with increasing complications and difficulties which stand in the way of our defining precise linear orders of the facts with which we deal. Our pleasures and pains, our comforts and discomforts, form to a certain extent series. The pleasure that one takes in food, in music, or in a particular conversation, may be greater or less, may [6][8] increase or decrease, and therefore one naturally comes to conceive of many of our pleasures as forming series of greater and less, and just so our pains too, our discomforts, are conceived as greater or less. But, on the other hand, we find it impossible to arrange all pleasures in a single row. It seems impossible to decide whether a particular moral discomfort is greater or less than a particular aesthetic discomfort. If one is asked to be quite sure where in the series of displeasures a given complicated discomfort stands, he may find great difficult in answering the question. A comparatively slight sentiment of remorse, say for what we consider a minor fault, -- is it or is it not greater than the chagrin of one who has made a social mistake, or who condemns himself for a failure to show the kind of knowledge on a given occasion he thinks he ought to show? Instances of this sort show us how hard it is to tell which of two discomforts is greater or less and to arrange all our pleasures and pains in a single series. For that very reason, however, the world of what we sometimes call values is a world especially hard to reduce to the sort of order that makes it the topic of scientific inquiry. There are many other ways of estimating values besides merely consulting our own conceptions of comfort or discomfort. The degree of social approval or disapproval with which our acts meet tend to form series. The public disapproval with which a given kind of fault is greeted seems to be greater than that which attends some other failure of duty, and in this way we get a kind of valuation series in terms of which many people try constantly to understand what the true value of their acts may be. But here again the complexity baffles us. The verdicts of society cannot be arranged in a single series. They are [7][9] not like the numbers, they are not like the tones, they are not like the division points on a scale. As the order grows more complex our knowledge tends to be baffled. So much for instances of simple serial order.

[8] Note at the top of the page reads, "Omit this page."

[9] Notes at the top of the page reads, "Read from the mark" and "Read the MS additional above mark."

But the instances that I have just been mentioning indicate to us that][10] in countless cases where we fail to reduce things to order by arranging them in a single series, we may seek to obtain an understanding of their nature by considering the relations that exist amongst various series of objects. As soon as I pass to this way in which we frequently endeavor to understand the natures of things, I at once come in sight of devices which are of the very greatest service in the science. These are the devices of what I call the correlation of series. Such devices consist in observing what facts occupy corresponding places in various series. Suppose that I am dealing with a number of objects that do not permit me to arrange them according as I say to any principle in a single series. Suppose that I can make two or more, sometimes an indefinitely great number, of relatively independent series of such objects. It still remains possible for me to understand these objects in their relations if I can connect one of these series with another or many of the series at once, in a definite way. The fact that I can do this is especially suggested to me by a familiar process upon which as a fact much of the practical and scientific value of serial order depends. If one asks the questions why are we so fond of arranging objects in series, why does it help us to find things in a row at all, the instance already cited of the numbered houses on the street easily furnishes at least one answer to the question. It is convenient that houses should be arranged in a single row along the street. It is convenient that definite numbers should be attached to the various houses, and why? Because hereby as one says it is easier to find a given house. One knows how to proceed from one house to another. One can look up the address of one's friend, and now in what direction [8] and how far to walk to find his house. But to mention this instance of the value of serial order is at once to remind us that in this case the value in question depends upon the fact that there are three distinct series of facts which in this case may be said to be correlated. My habits make it possible for me to walk along the street in a given direction. But this walking along the street in a given direction is of itself a series of acts. My habits make it possible for me to remember or to understand the order of the houses on the street. The practical importance of the orderly arrangement of the houses depends upon the fact that I can correlate the series of my wits in walking along the streets with the series of houses. Now the numbers form another familiar series which I can correlate both with my own acts and with the houses. This can answer the question as to whether I have to go further still in a given direction to reach a given number, or whether my walk will be a long one or a short one. Such an instance as this is one of the simplest cases of the process of the correlation of various series of which we find instances throughout the whole range of science. [In the library the books are arranged not only

[10] Marginal note beside the bracketed passage reads, "Omit."

on a single shelf, but on many shelves. One finds the single book not by considering its place in a single series, but its place in a very complex collection of series. One may have to go to a given hall, to a given alcove, to a given row, to a given shelf in that row, and look in a particular way for a book of a given number.

Examples of this sort introduce us to the vast realm of complex orders and most cases where we speak of order are instances where more than a single series of facts are concerned.][11] In this case a moment's thought enables us to say that we are helped in understanding the given order, first, by the fact that each series has its own order, and secondly, by the fact that the many series when taken together constitute [9][12] a single complex order. A complex order, however, consists in general of an interweaving or combination of various series, [and herewith we already have obtained a much more definite conception of what order is than the conception with which we began. Arrangement means in the first place serial arrangement. Complicated order means combination of various series.][13] Let us consider a very few instances of this type in order to supply ourselves with sufficiently varied examples of our general concept of order.

A family tree in a treatise on genealogy constitutes an excellent instance of a complex order. In such an order every descendant whose name stands well down in a series of generations can be connected with anyone of his ancestors by some perfectly definable series which you can read off if you will examine the tree. On the other hand, various series lead you as you examine the genealogy from one branch of a family to another, from one person to a very distant cousin, but meanwhile a glance serves to convince you that the tree as a whole is formed of a certain number of what you call fundamental series of facts arranged in a precise way. The tree for instance shows the descent from the ancestor who is regarded for the purpose of this genealogy as the beginning of the family. Several children of this ancestor are mentioned. Each one of these children has descendants and if you follow along any particular branch of the family you get a single series, but meanwhile, owing to the fact that there are many children of the same generation, the new series take their beginning in each generation and run side by side. Meanwhile when the order is all together before you you [sic] can yourself create as it were new series in the order that you find, and this is what you do if you ask how two very distant cousins are, as you say, connected. Here as you see a [10] complex order depends upon an arrangement of various series together.

Chronology as the historians develop it is a somewhat similar complex order by means of which historians conceive the various events that interest them with reference

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[11] Marginal note beside bracketed passage reads, "Omit to end of bracket."
[12] Note at the top the of page reads, "Omit from marked passage."
[13]Marginal note beside bracketed passage reads, "Read marked passage only. Omit to p. 14 mark."

to time. Here everything depends upon recognizing what they may call one fundamental natural series, the time series itself. The years follow one after another, and that series is a given fact behind which the historian does not attempt to go. To this series the historian correlates the various facts of any one historical sequence which interests him; the history of Egypt, the history of Greece, the history of China, the history of any state in Europe, may be thus considered and in such a history some series of notable events such as the births and deaths of kinds may be seized upon for examination. This series is first correlated with the series of years. Such processes having been accomplished in case of many series, one has then a chronological system of the history of the world or some portion of the world, and this system enables the historian to make as many new correlations as he pleases. So that before long one is considering how the events in Babylonia are correlated with those in Egypt, or how the kings of England are related to those of France. Here the entire science of chronology consists of serial, and therefore orderly, arrangement of facts. Such cases as the family tree and the historical chronology may make you suppose that complex orders of the type I am now considering have a merely formal interest. They enable one, you may say, to commit facts to memory, but do they assist one in the scientific understanding of the nature of things? Well of course even in the cases just mentioned, one's comprehension of the nature of things is certainly aided by knowing that two events are contemporaneous or that certain children were [11] the descendants of some common ancestor.

But we may turn from such formal instances to sciences that do indeed undertake not merely to report and arrange facts but to explain their laws. Consider the kind of order the astronomical science discovers in the heavens. Astronomical science began with observing the serial orders of the positions of single planets, and of heavenly bodies, such as the sun and moon. This simple series came in the course of time to be correlated so that one could predict what phase the moon would have on a given day; or when knowledge grew greater could discover on what day an eclipse of the moon might be expected, or even perhaps an eclipse of the sun. All astronomical knowledge of our own solar system has ever since depended upon our increasing knowledge of such series and of how they are correlated. But the heavenly bodies have been from a very early stage the very type to mankind of a world of order. It is very likely if we had been unable to see the heavenly bodies we should not have obtained the conception of natural law anywhere nearly as soon as we have obtained it. Modern astronomical science is now dealing with problems far more complex and difficult than those which the solar system presents. Now-a-days one is inquiring into the constitution of the whole stellar system, into the types of stars that are there present, into the laws that govern their appearance, their brightness, their physical constitution, their variations, their distribution, and even their whole evolution. But how does one proceed to acquire a knowledge on these vast and difficult topics? One learns to arrange facts about the stars in serial order and these serial orders grow more and more complex and more and more variously correlated as one proceeds. The stars vary in brightness. The modern astronomer deals amongst other [12] things with such questions as to how the stars of the various degrees of brightness, or as we commonly say, of the various magnitudes, are distributed. Are the stars of the fainter magnitudes very much more numerous than the stars of the higher magnitudes? Does the number of the stars of fainter magnitudes increase as rapidly as one would expect it to do in case the stars are somewhere nearly of the same actual size and are distributed in all parts of the heavens? Or do they cluster in certain ways and if so, according to what laws are they grouped? The physical constitution of the stars is in a great measure revealed to us by a study of their spectra. What series of spectra can spectroscopic examination of the stars reveal? Do all the stars belong to one type, or are there numerous types of physical constitution revealed by various types of spectra, and if in fact there are such various types, in what serial order can they be arranged and how shall this order be interpreted? What light does it throw if any upon the physical processes which led the stars to become what they now are? Certain stars vary in brightness, in more or less regular periods becoming more and more luminous and gradually fading away, or again, remaining steady for a long period and then for a brief period being almost extinguished. What are the various serial orders in which these phenomena occur? Again, as to these various stars, all of which are variable, do their ways of variation again form series so that some stars vary in one way, while others vary in another, and while still others lie between these two types? Upon the answer to such questions depends a knowledge with regard to the causes which make the stars vary in light and this knowledge, difficult as the problem is, is constantly growing. In a case like this you have an investigation relating to many of the most stupendous, remote, and [13] problematical facts of the physical world. The success of this investigation entirely depends upon the power of the astronomer to arrange observed facts in certain series. Some of these like the series of the brightnesses of the stars are so to speak forced upon him immediately by the observed fact that some stars are bright and others faint. Others of these series are so to speak much more artificial. For the different types of spectra that the stars when examined by a spectroscope show can be arranged in serial order only according to principles which the astronomer himself must more or less invent. Nevertheless whether the series are natural or artificial, upon their formation, their correlation, and the study of what results, the progress of modern astronomy depends and always will depend.

What the astronomer does in his world resembles after all what you find the chemist doing in his. A very slight study of chemistry will show you how important the conception of series is at every stage of the great science. The chemical elements themselves form a series or rather they can be arranged in a complex order which involves several related series. Upon the formation, the study, and interpretation of these series momentous problems of chemical theory depend. If one turns from such sciences in the apparently lifeless world to the world of natural history one still meets the same characteristics. Serial order is the first thing that a scientist seeks to establish. Correlations of series are the basis of all our higher knowledge of the connections of facts. Order means the existence of complex systems of interwoven series. Our scientific knowledge is a knowledge of the relations of such series. In terms of them we find facts, we know how to name them, we know [14][14] how to find them again when we need them, we learn how to measure them, we learn how to discover the conditions which determine the variations to which facts are subject. Every law with which we are acquainted, if it is an exact law at all, relates to the serial order of facts or to the connections amongst series, and this I say is as true in the world of living forms as it is in the world of the stars or of the chemical elements. *I mentioned in the early part of this lecture the way in which a geologist views the facts that have to do with rock formations and mountains. A very slight acquaintance with geology shows you that the entire science depends upon discovering the serial order of certain facts. The geologist has his kind of chronology. One formation succeeds another. Two different formations in different parts of the world are more or less exactly contemporaneous or at any rate belong to the same general geological period. Here as you see you have on one hand the series of geological formations and on the other hand the correlations of various series as they occur in different parts of the world. Every explanation of geological formations depends upon the establishment of a similar series. If we were to pass from the world of natural history back again nearer to the practical world, to the world of social affairs, you would again see how the establishment of serial order among facts is the key to every exact comprehension that we have of the connections of facts, in society as in inorganic nature. You would also see how our inability to discover certain series which we should like to be able to know in our study of social phenomena limits our power to comprehend the workings of society.

[14] Note at the top of the page reads, "Read from the bracket."

But I will not weary you here with further illustration. We set out to find out what order is. We have found that order in its [15][15] simplest form consists in the existence of certain linear series such as the series of numbers or the series of the division points on a scale. We have found that all more complex instances of order are complications made up out of series. We have already seen in a measure why serial order is so important. It is because in so far as we conceive facts in a series we are able to find the single fact in its place and to understand definitely its relation to other facts. We have further seen that our knowledge of the laws of nature is very largely dependent upon our power to correlate series. That is, to find out what fact in a given series is corresponds [sic] in a given way with facts in some other series or as I should say, is on a level with these other facts. And so then we may say to reduce things to order is to arrange them serially and to apply similar processes to the arrangement of these series so that out of various series a complex but still orderly system results. We see that in as far as we have science we succeed in arranging our ideas of things in this way in our minds. We see that nature helps us to form these series of ours by presenting to us in a large number of cases facts that as it were come to us already arranged in series. But we see that in many other cases nature does not come in this way to our aid, but on the contrary furnishes to us facts that appear disorderly. In such cases our understanding depends upon our power to form artificial series which still more or less represent the true nature of things. We then find that all our further power to comprehend the order of things depends upon our skill in bringing into relation these various series, in correlating them, in linking one part of one series to a given part of another series according to some definite principle.

Science then has this in common with housekeeping, with the [16][16] arrangement of dinner parties, with the orderly conduct of a railway business, with the activities of the courts of law, with all in fact that is rational about our lives, science mainly depends upon finding or making serial order.

But the generalization which we have thus far made is after all not very enlightening unless we are able to understand a little better what we mean by a series itself. I spoke of the series of numbers. I have spoken of the series of the dividing points of a scale. I have mentioned numerous other cases of what we call series of facts, but just as I before asked what is order, and have now answered order is arrangement in a series or in systems of series, so now I must press the question further and with you ask what constitutes a series? By series one plainly means a collection of objects. A single object forms no series. Two objects can hardly be said to constitute a series.

[15] Note at the top of the page reads, “Read.”
[16] Note at the top of the page reads, “Read.”

Three objects you can arrange in series and any greater number may constitute a series. But a series is a particular kind of collection. It is a collection that is obviously characterized by something very much like a law. To be able to count already tells me something about the laws of numbers. It is very common to speak of the law of a series. Nevertheless the term "law" is somewhat too broad and vague a term to assist us in comprehending just what is meant by that arrangement of objects which constitutes a series. Because the matter is decidedly important for our subsequent understanding of the principles and the meaning of science, I must venture to trouble you as I close this discussion with a few very highly technical considerations, simple, but like most logical matter at first sight, very dry and uninviting. [17][17]

Finally the objects which constitute a series are in some notable kind of relation to one another. To understand what I mean by series depends then upon a still more fundamental conception. It depends upon understanding what I mean by a relation. Now nothing is more familiar to the intelligent mind than the relations of things. Your family life depends upon relations. Relation of brother and sister, of parent and child, of husband and wife. All commercial and social life involve relations. Every form of exact knowledge is a knowledge of relationships. But if again you ask me what relation is, I find it difficult to tell, although nothing seems more familiar if you do not ask me. I will not attempt in the present discussion to answer all the philosophical questions that may be raised about the nature of a relation, but this I may at once say: a relation considered as the relation of anything you please to anything else, a relation such as that of a brother to a brother, of a debtor to a creditor, of an equal to an equal, of a king to a subject,-- a relation, I say, is the character that a thing possesses when that thing belongs to a certain collection and where the thing loses it in so far as the thing can get out of that collection. Thus, a member of a club has a certain relation to the other club members. That relation belongs to him as long as he belongs to the club. If he leaves the club the relation lapses. The king has a certain relation to his subjects. It exists because the king and his subjects form a particular kind of collection. If the subjects rebel or vanish the king loses his character,-- no longer has his relation,-- ceases to be a king. The fact that objects can possess certain characters when they are in certain collections and lose these characters when [18][18] they leave the collections is a fact that for the time being we shall regard in this discussion as fundamental. So much then about relations in general. Now as relations are a very familiar fact so they are in a sense a vast realm of very various facts. It seems at first sight hopeless to try to classify them. Yet for our present purpose there is one classification of relations that it is

[17] Note at the top of the page reads, "Read."
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necessary for me to make, because if we understand that classification we shall be able to understand better the nature of series, and so better the whole conception of order, and in the end we shall be able to understand better why orderly arrangement is so vital, both theoretical and practical. Relations might first be classified according to the size of the collections in which they hold. [Thus obviously a king needs in general a great many subjects, and one who declared that he was a king, but who lived on a desert island with only a single subject, would not appear to us to be worthy of the name. Exactly how many subjects a king must have in order to be a king, we cannot say, but kingship is a relationship that involves a decidedly large collection. Instances of similar and more definite collections with attending relationships we see in the case of such collections as a baseball nine. The pitcher of the nine stands in a definite relation to every other member of the nine. His position as pitcher then determines a system of relations which belong to him in this collection, and which he would lose if he were no longer a member of the nine; but the relations themselves belong to him as a member of a certain group consisting of the nine itself and including perhaps the substitutes.][19] The size of this collection is limited. Some relations are characteristic of collections of three objects. Thus, if I tell you that [19][20] on a bench there is one man who is between two other men, you observe that the property of being between is a property that belongs to one member of a collection of three. But the most familiar instances of relationship are characters that an object possesses as a member of a pair of objects. Some people are fond of limiting the use of the terms to this case. As a fact such limitation is unfair, yet on the other hand, pairs of objects present certain relationships which are extremely familiar and which especially interest us in ordinary life. Thus when I say that A is in front of B, I mention a character that A possesses as a member of a pair which pair consists of A and B. Similarly, when I speak of A as a debtor of B or as a lover of B, I mention each time a relation which exists between the two objects, or rather a character which belongs to one of them as a member of this pair of objects. Now such relations existing within a pair of objects only sometimes get the technical name of dyadic relations, and as I said, dyadic relations are a most familiar one and most such relation names as friend, lover, father, servant, debtor, creditor, enemy, etc., are names for such relations. Let us hereupon fix our attention upon these dyadic relationships and observe how they may be classified. There is one classification which has great technical interest and also a very obvious practical interest. If two objects constitute a pair, they may be on even terms as members of the pair or they may be in some sense unequal members of this pair. Thus if you are speaking of the world

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[19] Marginal note beside bracketed section says, "Omit."
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of man, and you say that A is the brother of B, then at once in this pair B has the same character that A has; B is a brother of A, and that is the nature of the relation. It is what one may also call a reciprocal relation. It is [20][21] a relation such that if one of the members of the pair has it, the other also has it in the same sense. Again, if for any reason you say that A is the equal of B, you say that B is the equal of A. The relation of equality is reciprocal or mutual. But there are dyadic relations which are distinctly not mutual or reciprocal. Thus if A is the debtor of B, B is in so far of course the creditor of A, and the relations of debtor and creditor have a sharp contrast. If A is the superior of B, B is the inferior of A. If A is greater than B, B is less than A. Now relations of this sort, relations in respect of which the two members of the pair in question are not on the same level, are not equal, are called non-reciprocal relations. There is another and more technical way of naming the distinctions between these relations which we shall here find convenient. The relations of the first type which we have called the reciprocal relations are often and conveniently denominated symmetrical relations. Such I repeat are the relation of brother, friend, equal, companion. You must observe at once that these relations have a certain practical interesting character. They are relations of comity. They are the relations of the type which we sometimes have in mind when we speak of fair play, or of even justice, or of something of that kind. Nevertheless these relations are of very great importance as you shall see in regions of thought to which such practical considerations do not apply. The relations of the other type are called unsymmetrical. They are, I repeat, such relations as debtor and creditor, greater and less, higher and lower, father and son, etc.

So much for a very simple classification of dyadic relations which in a moment we shall find very important for the purpose of understanding the nature of series. I now am obliged to trouble you with [21][22] another classification of relations. A relation may hold within a pair of objects and there may be another pair of objects within which the same relation again holds, and now these two pairs of objects may have a member in common. Thus for instance A may be the friend of B, B may be the friend of C. Again A may be the equal of B, and B may be the equal of C. Or A may be the debtor of B and B may be the debtor of C. Whenever such a situation appears the relations in questions may have one or another of two very markedly distinct characters. Consider for instance the relation of equality. As they say, things equal to the same thing are equal to each other. Hence, if A is the equal of B, and B is the equal of C, it follows inevitably that A is the equal of C. The relation of equality is then one which enables me to do what Professor James has called "skipping intermediaries." When I once

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[22] Note at the top of the page reads, "Read."

know that A is the equal of B, and B is the equal of C, I can skip B altogether and say at once that A is the equal of C. Now there are a great many relations of this type. The relation of greater and less for instance is of this type. If A is greater than B, and B is greater than C, A, B, and C being for instance quantities, then A is greater than C. I can here skip the intermediary although I am no longer dealing with the relation of equality. If B is to the right of A, and C is to the right of B on the line, then C is to the right of A, and the relation of "to the right of" is again a relation which enables me to skip the intermediary. If B is in front of A, and C is in front of B, then C is somewhat remotely of course, but still really in front of A, and here again is a relation which enables me to skip an intermediary. Relations of this sort have a technical name; we call them transitive relations. They are relations which enable us to skip intermediaries [22][23] and to proceed to certain conclusions. You will observe at once that such relations can be themselves either symmetrical or unsymmetrical. Thus, equality is a symmetrical relation, but since it permits me to skip an intermediary and the reason that if A etc., equality is also a transitive relation. On the other hand, the relation of greater and less is unsymmetrical, but it is transitive, and in so far is like the relation of equality. But some relations, both symmetrical and unsymmetrical are not of the transitive type. Thus, if A is the father of B, and B is the father of C, the relation of fatherhood proves to be not transitive for then A is not the father but the grandfather of C. On the other hand, the relation of ancestor and descendant is transitive. If A is an ancestor of B, and B is an ancestor of C, then A is an ancestor of C.

Herewith we reach another classification of our relations. Some of them are transitive, and some of them are intransitive. This classification again has a very great practical importance. Upon transitive relations depends our whole power to arrange our world in linkage, as it were, so that by intermediate steps we can pass from one thing to another. Every chain or rope is an example of a sort of transitive relationships [sic]. If you pull the end you pull the middle, and thereby you pull the other end. If you move one link of a chain you move the next, and so on to the end of the chain. The locomotive pulls the last car. Wherever many things are linked they are so by transitive relationships, or at any rate, by relationships which can be interpreted as transitive dyadic relationships. On the other hand, relations which are not transitive help to break up the [23][24] world into disconnected groups, to separate things, to keep them apart. Some relations are so to speak intermediate between the two types, being sometimes transitive and sometimes intransitive. The relation of friendship is an instance of this sort. If A is a friend of B, and B is a friend of C, that tends to establish a reasonable

[23] Note at the top of the page reads, "Read."
[24] Note at the top of the page reads, "Read."

probability that A under favorable conditions should become the friend of C, and the relation there is a fact frequently proved transitive. On the other hand, as we know, the transition is by no means always made.

Well so much for our two classifications of relations; symmetrical and unsymmetrical on the one hand, transitive and intransitive on the other. But now in mentioning all these cases I have been intending to prepare the way for an understanding of the nature of series. How can I accomplish this? I said in beginning a series is a collection of objects. I now add, wherever there is a series there is a single transitive and unsymmetrical relation which links together the members of this series in such wise that if you choose any two of them, one of them stands to the other in this transitive unsymmetrical relation. Thus, a row of men constitutes a series. There is a transitive unsymmetrical relation. The relations of before or after. If you choose any two men of this row, one of them is before the other, and the fact that this same relation binds any pair in such wise that one of the pair is before the other enables you to arrange all the members in the series in a single row. Precisely so with the numbers. There is the relation usually of greater and less. It binds any two numbers together in such wise that one of them is greater than the other, and that fact makes the numbers stand in one series. Now I say that wherever we have a series we have such a transitive, unsymmetrical relation which binds any pair of things that [24][25] belong to the same series. This relation remains constant throughout the series. It is a relation upon which the series depends.

But now let us glance at what happens when we correlate or link together many series which belong to a single system. And in order to see what then happens let us turn our attention once more to the relations of the other type that I mentioned, the symmetrical relations. Symmetrical relations I said may be transitive, so the relation of equality is transitive. Now you will observe at once that if I have a large collection of objects of which I merely know that any two of those objects are equal to each other in some respect,-- in size, in weight, or in capacity, or whatever you please,-- then those objects are indeed set in no sort of order by this fact. Thus a collection of weights, all of the same size, do not so far constitute an order, whereas if I had a set of weights that could be arranged in a single series that would be because some such relation as greater and less held between any two of them. If I have then a collection of objects any two of which are bound together by a single symmetrical transitive relation, such as equality, these objects do not of themselves constitute an order, but such a set of objects may form a very important part of an ordered system. Of course when I correlate or bring into connection two or more series of objects I do so by establishing symmetrical

[25] Note at the top of the page reads, “Read.”

relations amongst the members of different series. Thus when you get a train to run on time, you establish certain symmetrical relations between the train and any station that you please on the road. Namely, the train and the station are together at some determinate moment of time, but the relation of being together is a symmetrical relation. So correlation depends in general upon symmetrical relations. These relations may be transitive, and in that case they enable me to correlate a great number of series. Now when I look back upon any of these complex instances of order which I [25] discussed earlier in this lecture, you will see if you examine these cases carefully that in all of them the facts with which one deals are either arranged in series or else are in places which in various series correspond, so that they are in some wise correlated together. And now let me define one more term. When a collection of objects is such as to be bound together by a single symmetrical transitive relation, I call this collection of objects a level. Thus the things that are in two pans of an apothecary's scales are on a level if the scale pans balance. The places that are on a level with the sea are in a symmetrical transitive relation to one another. When in a large social order a great number of men are legally or socially equal one to another, these individuals constitute a level. And now if you look over the various orders that I have spoken of, you see that what we do when we deal with such orders in so far as we consider the correlation of various series is to arrange various objects on the same level. Thus while levels themselves do not directly put things into order they help us to bring various orders into correlation with one another.

We began our discussions with the inquiry, what do we mean by order? We have found that order consists in the existence and correlation of series. We have discovered that where various series are correlated we have what may be called levels. We have discovered that the arrangement of things in series has both a practical and theoretical importance. We have seen that both the formation and correlation of series depend upon the establishment of certain relations which again have both a theoretical and practical importance. These relations are of two types: [26] the symmetrical transitive relation, such as equality, the unsymmetrical transitive relation, such as greater or less. We have seen that these relation types have their practical importance. What we shall have further to see at our next lecture is the way in which these relations and the orders which result from them help us to an appreciation of the laws of nature.

Lecture III: Instances of laws, series, levels[26]

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The term law is used in a very great variety of senses and one sometimes feels that such usage has made it so vague that one would prefer not to employ it in designations that pretend to exactness. Still the word is once for all there, and it is supposed to be of great importance to us to become sure that nature is subject to law. We ask the question with which these discussions opened, namely what is the general lesson that the study of science has taught us and the most familiar answer is that we have learned from the study of science how nature is everywhere a realm of law. Since in many cases the laws of nature as discovered by the special scientists do not seem to have any very close relations to our own ideals and human interest, it is not infrequently asserted that when we study science we come to deal with a world that seems hard and cold and opposed to the aspirations of the heart. Yet precisely as we have found that the arrangement of objects in series and the grouping of other objects upon the same level with respect to a given series is a process that expresses human activities and that when accomplished involves the successes of human efforts, so we have now to see that every time that a human mind leads to the successful issue. Whatever ideals a man has to give up when he studies science, he certainly wins something in return for them. The view of the world that science gives us is everywhere a rational view, and in so far the ideal view. We are interested in finding law in our world of experience. What I wish to say to you on the present occasion is what interest the search for law in experience meets and what ideals we accomplish when we discover that law exists. To this end I must begin with somewhat vaguer and less definite conceptions of law and then quickly pass to those instances [2] which permit a sharper characterization of what it is that we look for when we seek for law, and what we find when we discover law. In the vaguest sense I know something that I can call a law whenever I can make with confidence a general assertion. Whenever I can say that all men are mortal or that all summer days are long, I may be said to be acquainted with a law, but the law that appears in this form is not one that stands for any very highly developed type of scientific knowledge. Even within the range of common-sense we are not content to know laws of this type and these alone. When you go shopping you are not content to be certain that everything that you buy must be paid for. You need to know the various prices of various things. In general, if you are considering what to purchase, you arrange various purchases and prices in series, or again upon a level. You consider that this object will cost more than that, or that any

[26] This title, too, was added by E.F. Wells in red pencil (cf. note 2).

one of these objects may be bought for a given price. Only when you have a considerable number of assertions of this sort in your possession, only in other words when you have reduced the prices of possible purchases to order, have arranged facts in series and upon levels,-- only then can you say that your shopping is conducted according to some kind of system, and that your knowledge of the present laws of the market is a knowledge that approaches exactness. Well now what holds in the world of common-sense holds also in the world of science. Propositions of the form, all A is B, and therefore if A occurs then B occurs, are frequent propositions in science, but no one such assertion is enough to constitute a scientific insight into the connections of facts. We know that all men are mortal, but such knowledge is not scientific. The knowledge that an insurance company possesses about the average percentage of the healthy men of a given age who will die within ten years is a knowledge that in its own way [3] approaches the scientific type. It is knowledge of what we call the statistical kind, and knowledge of this sort always relates to series of facts, and not merely to single classes of facts. The life tables show what proportion of a given class of men may be expected to die within a year, within two years, within three years, or within ten years. And the use of mortality tables depends entirely upon the construction of such series and of the various levels that go with them. Our knowledge of the laws that relate to the consumption of fuel is a very good instance of the sort of knowledge of law that really interests us in dealing with nature. To know that whenever you want to run an engine you must burn coal is not yet to know very significant laws of nature. The question that you desire to answer when you have to deal with a particular engine is the amount of fuel that you will use in a given time in case you are to get a given amount of work from that engine under the given conditions. Knowledge of this sort is inevitably related to a knowledge of facts that form a series. If you burn so many tons of coal you will get so much work done. A given increase in the amount of coal burned will produce a given increase in the amount of work to be done, and so on. Such knowledge becomes at once valuable for deciding how much coal must be put into a given ship in order that she may make a certain voyage. Again, how much coal must be purchased by the manufacturer in order that certain goods shall be turned out? You see then that it is almost useless to say that when we study the laws of nature we are endeavoring to find out merely general propositions. Our various general assertions must be assertions about objects that form a series, or again, about the relations of various series to one another. I may put the same consideration in another form. Textbooks of inductive [4] logic, that is, of the logic of science, very frequently tell you that the purpose of scientific investigation is the discovery of causes. Now a cause, such books say, in physical science, is an antecedent such that if this antecedent is present in a given case a certain consequence will follow. Examples of a knowledge of causes one finds in the textbooks in a case like this: An inquiry arises as to why a feather and a bullet fall with unequal velocities when they are dropped in the air, the bullet coming at once to the earth, the feather floating slowly down. The hypothesis arises that it is the resistance of the air which causes the feather to descend slowly. This hypothesis is tested by a well known experiment,-- the experiment of permitting the bullet and feather to fall in an exhausted receiver. The air is here to a very great extent eliminated. The bullet and the feather fall in almost exactly the same time. And so one concludes that the air is indeed the cause of the ordinary difference in the behavior of the two bodies. Such, say the textbooks, is scientific knowledge. I do not deny its importance as such a fragment. What I wish to point out is, however, that physical science does not merely consist of setting side by side a large number of essentially fragmentary assertions like this one. Physical science is interested in quite another aspect of such laws besides the aspect that can be expressed in a single general proposition.

John Stuart Mill in his logic laid considerable stress upon a method of investigating nature which he called the method of concomitant variations. You wish to find whether the air causes the various bodies to fall with different velocity. Accordingly you study various cases where the air has been partially taken out of a receiver, where the air has been still more completely taken out of a receiver, and so [5] on until you come to the case of a receiver as nearly exhausted as for the purposes of your experiment it is possible or convenient to exhaust it. In such a case according to Mill you reason that whatever antecedent varies in such wise as to be attended by variations of a given consequent must be a cause or at least part of the cause of that consequent. Here one speaks as if the study of a great number of cases where a given supposed cause varies in amount, has as its principle object the discovery of the one proposition that the cause which thus varies must indeed be the cause of that effect which is found always varying amongst the consequences present in the various instances. Similar is the reasoning according to such account when one notices that heat expands a large number of substances. One finds for instance that the more one heats a bar of iron the longer the bar grows, and that similar concomitant variations are observable with a great many different kinds of substances. Mill's treatment of the method of concomitant variations is usually interpreted by the student as if the principal lesson of it were that in view of these manifold parallelisms between the variation of a certain cause and the variation of a certain effect, we can generalize and say this cause is indeed the producer of this effect. I do not suppose that Mill regarded such a generalization as the sole or as the principal significance of his method of concomitant variations. What I wish to point out here is, however, that as that method is very generally interpreted by students the essential point of such investigations is missed. What interests us in case we discover that certain variations of A are attended by certain variations of B is not the mere generalization which under such circumstances may become very [6] soon a common-place, namely the generalization that A is at least a part of the cause of B. What interests us is the correlation of two series of facts. The various stages, degrees, amounts of A, and the various resulting amounts of B. You may know some such generalization as the socalled [sic] law of the relation of demand and supply. You may know that if the beef supply in Chicago is hindered by strikes or by combination the price of beef in your market bill will rise. But the interest of the householder is not in this generalization. What he wants to know is how the price of beef varies. In other words, he wants to know such series of facts. If he becomes interested in tracing the causes of these facts, his interest will inevitably be to discover if possible how the variation of the supply of beef in Chicago is related to the price of beef in his market bill. He can discover this only in case he can correlate two series of facts.

And so now I say our knowledge of nature assumes the form of exact law precisely in so far as we know how various series are correlated together. So much for our first view of what we mean by knowing the laws of nature. It is because of this interest of ours in correlating various series of objects that the processes of measurement become of such great importance for scientific work. Exact science is possible, so we are often told, solely because of measurable facts. This assertion is logically speaking not universally true. But the extent to which it appears to be true is mainly due to the fact that whenever we are capable of measuring objects we are capable of arranging measurable objects in perfectly definite series. Assume for instance that I am able to weigh the amount of coal in a ship's bunkers or the amount consumed in a given time, it is possible to correlate the consumption of coal with the length of the voyage, with [7] the speed of the ship, or with some other such fact, this other fact itself being capable of measurement, so that different speeds of the ship, or different lengths of voyage may be exactly correlated with given amounts of coal burned. There is then nothing magical about the importance of quantity in science. The value of quantities in measurable objects generally is that they can easily be arranged in series and the consequent various series can be correlated together. But quantities are not the only objects that can be arranged in series. The points on a line constitute a very definable series of objects, although the points themselves are not measurable quantities, and although it is possible to consider their relative positions without measuring lengths. The fact of a highly exact science of a branch of geometry is possible, and exists, wherein no sort of measurement is used, and one is dealing merely with the relative positions of various points. This branch of geometry is precisely exact as that popular and more familiar branch of geometry which turns upon measurement. In what is called non-metrical geometry one correlates various series of points; declares for instance that the points, a, b, c, and d on a line form a particular kind of row with b between a and c, and c between b and d. One further proceeds to prove that such socalled [sic] ordinal relationships amongst the various points remain unchanged when you pass from a given line to the shadow or projection of that line on a given plane. Such geometry continues by making assertions with regard to whether two lines have a point in common, and what point they have in common. A prodigiously exact science can thus exist in which nothing is measure, but everything is ordinally [sic] determined and the whole system of facts with which one is dealing constitutes an exact order with precise series and definite levels by means of which [8] definite series are correlated. I can conceive an absolutely exact moral science, remote enough from our present inadequate knowledge of our moral relationships, but such as some higher being might have. This exact science would determine precisely what act rightly or justly corresponds with a given situation. The laws of such acts would be laws that you would have to express in the form of complex series interwoven in systems. But neither acts nor their consequences, nor the situations in which they are required would need to be measured. Everything would have to be reduced to order. And everything would be reduced to order by means of establishing certain series. Thus, one would arrange possible morally significant situations in series. One would arrange corresponding acts in series. One would declare by a correlation of these series what act or acts properly corresponded to definable situations. And one would express all these correlations in terms of moral law; being in possession of such a complete moral system of the universe, would know precisely what to do in every possible situation. He would have an exact moral science, but he would not be measuring anything. The general relation of our conception of law to our conception of serial orders of facts having thus been suggested, I shall now try to proceed to a somewhat more detailed statement of what happens when we try to reduce our knowledge to scientific order with respect to the laws of nature.

In order to proceed further in this inquiry I have to introduce yet one more technical word. It is the word that you will have seen in my title, the term transformation. By transformation I mean the same as any definable change. A mere change may seem to us very unintelligible, in case it is sufficiently novel or violent or rapid. Thus when a flash of lightening comes it may be hard for us to tell [9] precisely how the flash seemed to be related to ourselves, what it direction [sic] it came from, and so on. A sudden and violent physical pain seems to us surprising, but unless the pain lasts some time, we find it hard to describe at what point the pain began. A patient at the outset of a given illness whose nature is unknown to him finds it difficult to give an account of his own symptoms to his physician. If for the first time you feel a Leyden jar shock your impression is that something of great importance has happened but you find it very difficult to say what has happened, except in so far as you are sure that you do not wish to repeat that experience. When the dentist pulls a tooth a similarly unintelligible change of one's state of consciousness occurs. It arouses tremendous repugnance, but resists analysis. On the other hand by a transformation I mean a change in so far as it is conceivable, definable, analyzable. If I pick up this object and remove it from this place to that place, a change not merely occurs but I observe what it is. A journey made according to a plan constitutes a definable series of transformations. A series of events constitutes a set of transformations precisely in so far as the historian can tell us what happened. Now it is obvious that all our knowledge of the laws of nature is a knowledge that has to do with the transformations of nature. The world of our experience is a realm where phenomena are constantly shifting, so that new things get substituted for old. So far as our knowledge of these changes remains vague, indescribable, indefinite, the changes may be real, but we do not conceive of them as definite transformations. The changes of the weather, vaguely noted by our passing consciousness do not in so far constitute scientifically definable transformations. But the Weather Bureau, taking hourly observations, defines the transformations [10] of weather that occur in the country during the day in a precise order. And of course all scientific knowledge of the weather must be obtained by such exact analyses of transformations.

And now when we study nature, of necessity we are not interested in single transformations, but in long series of transformations. That at ten o'clock in the morning of a given day the temperature in a given place rapidly fell – this is a transformation which may for a given purpose be regarded as a single transformation. A knowledge of the laws of the weather depends upon knowing long series of such transformations. Here again serial knowledge inevitably proceeds and lies at the basis of all the study of the laws of things. But what constitutes a single series of transformations? Anything that happens in the universe is succeeded shortly afterwards by something which happens, it may be in a far distant place in the universe. Thus every event may concern entirely different objects or people. Thus the eruption of Krakatoa on the earth may have been followed some time afterwards by an event which occurred on the moon or some distant star. It seems inconvenient to arrange such apparently miscellaneous transformations in a single series. Even historians whose reports of events have often to be very miscellaneous are anxious to reduce their narrative to order by putting in a single series of events which as they say belong together and so constitute a single set of transformations. A chapter of the history of France may be put in a general history by itself. It may then be followed by a chapter relating to some nearly contemporaneous events in the history of England. And so one may proceed from country to country. However, when one's facts are arranged there is a strong interest in regarding certain facts as belonging to one series and certain other [11] facts as belonging to another series of definable changes or transformations of the natural world or of human experience.

If we ask now what principle determines us to call a given set of events, events in some one series, and to regard certain other events as belonging a different series [sic], the answer is not difficult to give. A single set or series of transformations always means for us a set or series of transformations which appears to us to leave something constant, something changeless throughout the series. We estimate transformations by contrast with permanence. In a world when anything was followed by anything, and where not the least constancy could be observed, we should have no reason to arrange transformations in serial order, or to conceive them as constituting connected systems of events. The series of changes which take place when the steamer during the voyage consumes her coal is a single series in so far as despite the changes in the coal something remains constant throughout the voyage, namely, the ship, her destination, and whatever else it is in terms of which we recognize the series of incidents in question. A series of events in the history of France is characterized by the fact that even if this series occurred during the French Revolution when tremendous changes were in question, something, namely, the French people, remained in some notable respects the same, throughout that series of changes. A historical narrative whose scene and people change with every new event mentioned would have no sort of unity. In short, the unity of a series of events is definite in what remains permanent to that series of events. So far we have in our possession these results; that our knowledge of the laws of nature is a knowledge which has to do with series of events; that a series of events is defined as a single series by [12] virtue of something that remains constant throughout that series; while our knowledge of the laws in question depends upon the correlation of various such series. Having premised these considerations, I may now proceed to assert in general that when one knows a law of nature one is aware that a certain set of changes or transformations forming a series is such as to leave unchanged or unaltered certain definable conditions. If the law of nature that one knows is a complex and important law, one's knowledge goes further. It then consists in ascertaining that a great many different series of events are such that the type of transformation present in all of them leaves unchanged a particular type or character present in the things which you are considering. The statement now made becomes by itself too abstract, and must at once be illustrated. When the officials in charge of navy supplies know such laws regarding the consumption of coal as enable them to determine what supply each ship should have for a given purpose, the laws that they know relate to some such complicated relations as the following: If any ship sails she will burn coal. The series of events consisting of the tons of coal burned by a given vessel on a given day, will determine the time which will elapse before all the coal in the bunkers is consumed. Now the officials who determine the supplies of the navy will be able to predict for a given ship within limits how much coal she will burn during a given voyage. By virtue of what knowledge of law are they able to make this prediction? By virtue I say of their knowledge that a great many different series of events, each one of which consists in the burning of coal by some one ship. Now a series that have certain characters in common; that is, if you pass from one such series to another, you find that the coal burned has a certain [13] constant relation to the work done. This constant relation remains unchanged no matter how many different ships and how many different voyages you are considering. Or if it does not remain actually unchanged, that is precisely because the laws of the process in question are not fully known and the influences which determine the consumption of coal are so complicated, including the habits of engineers and coal heavers, that you are unable to say precisely what will be always the amount in case of all the voyages and all the ships. Supposing the loss to be known, the laws always take the same monotonous form. Consider a great many series of events. In each one of these series something remains constant. If you pass from one series to another this contrast will itself change, as the ship that is engaged in one voyage is different from the ship engaged in another and the coal used is different. But still as you pass from series to series something will still remain unchanged. If you correlate such series as the ships burning their coal with such series as stationary engines on land burning their coal, and finally with such series as the student of the theory of heat considers in general, you get still wider and wider generalizations. They all have the type that great numbers of series are in such wise correlated that this or that remains unchanged as you pass from series to series. Since your passing from series to series constitutes itself a more or less ideal set of transformations, you reach the final result that all our knowledge of law assumes this form, that certain series of changes, real or ideal, constitute such a system that a definable something remains unchanged throughout these series. The ideas of which I am dealing are here decidedly technical, but the illustrations are simply countless. Let us exemplify the whole process is still another way. A socially [14] skillful person knows how to behave in various company. This knowledge he has with regard to the proper behavior is of course founded upon a knowledge of the laws of such social groups as he is accustomed to associate with. Now any occasion when a man meets a company of people presents to him a set of transformations. The people speak, move about, co-operate, come and go. These transformations on any given social occasion are such as to leave unchanged certain conditions, namely the conditions upon which civilized society in general, and this social occasion in particular, may depend. The socially skillful person now not only follows any such series of events with success, but when he passes from social occasion to social occasion, when he meets now this company of people, and now that, he knows how to behave and to succeed on each new occasion. But in so far as he is aware of his skill, in so far as he knows upon what his success depends, his knowledge consists in an acquaintance with certain principles of conduct which remain unchanged whatever the company or whatever the social occasion. Such a knowledge of the laws of social occasions is of course very insignificant. It is practically liable to all sorts of error. With most socially skillful persons a great deal of such knowledge remains unconscious. Every social life consists in so much of definable transformations, as of more or less fascinating, surprising, or appaling [sic] changes in the social environment. But so far as such knowledge is exact, is conscious, is such as to lead to success, it is always a knowledge of what relations, character, principles, modes of behavior,-- the transformations of the social order always leave unchanged. The socially successful man who comes into a new social group has in so far very greatly transformed his social situation. He no longer is where he was or amongst such [15] people as he has been meeting in the past. So far as he still adjusts his conduct to the situation and knows why he does so, he keeps on following such principles as he has already established. In other words, the very novelty of his conduct, the skill with which he adjusts himself to a new situation, depends upon preserving intact exactly those principles of conduct by which he succeeds. He acts well this time upon this novel occasion because he has rules that he can hold constant, while these rules are precisely such as determine what he has to do upon this occasion. So here again the knowledge of law is the knowledge of what remains unchanged amid transformations.

I have thus defined the nature of the knowledge of law by first speaking of the series of facts whose laws we discover and by then pointing out that our knowledge of law is the knowledge of what some series of changes inevitably leaves unchanged. I could approach the whole subject in another way and repeating considerations which occurred to us at the outset I could illustrate how no knowledge of what is changeless in the world is of the least service to us unless it at once is related in our mind to the definition of a series of objects, and in fact to the definition of a series of changes. What is it that always happens in the world? I can of course answer life always goes on, the sun always shines, men die daily, in brief, I can utter any common-place that you please about life and the world. In so far I do not name any notable law which is of real scientific importance. But as soon as I say this time when you are preparing coal for this voyage you need so and so much coal, because this case has a definite place in a series of possible cases, a series consisting of different possible ships with different possible voyages and different possible amounts of coal needed,-- then and then only do I make use of a definite knowledge of the law. But my knowledge of law [16] now takes the form of saying that since all the members of this series are of a certain type, this particular character, just in order that it may conform to the general type. In the series of whole numbers, each new whole number is a single case. But of course the whole series is formed according to a certain law, and all the whole numbers conform to the type of this law. This whole number in its own place in the series must have some special character in order that it may here illustrate the general law. The general law of the whole number is that each one follows a particular number and precedes a particular number. In order that seven for instance should obey this law, it must follow six. And as the follower of six it has to have special character which results from its place in the series. Hence, series and laws are inevitably correlated. If I begin with a series, it becomes of scientific importance to me in so far as I observe some law, it either holds in a single series or holds when I pass from series to series. On the other hand, so far as I define law I define what is a mere common place unless I apply it to the series of facts and so determine what this fact must be in its place in the series, because of a general law that is verified by all the facts that form the series.

So much for a sketch of the way in which laws and series go together in our knowledge of nature. But now for one or two illustrations of how the notion which we defined at the last time, the notion of a level, is related to our conception of a law of nature. When I set a great number of series side by side in my mind, and so correlate them that certain facts belonging to these various series occupy a corresponding place in all the series, then as I said at the [17] last time the facts in question lie on what one may call a level. Thus, if I correlate the series of facts that I meet with ascending one mountain with the series of facts that I meet with in ascending another mountain, one very convenient way of carrying out this process is furnished me by the familiar process of defining how high above sea level any given point on any given mountain is. Now all the places on the earth's surface that agree in being 1000 feet above sea level, are themselves upon a certain level. This level may determine certain similar constant characteristics which I can verify from place to place on the earth's surface. Thus for instance within limits the air pressure at a given level tends to have the same relation to the air pressure at sea level at the same moment whatever part of the world I may happen to be in. In this way, the correlation of the various elevations above sea level is carried out by what one may call a series of levels. Since wherever we correlate various series the same considerations come into sight, we may observe that most, if not all, instances where we speak of laws of nature, can be expressed by saying that various facts in various real or ideal series stand upon the same level. For as we have thus seen, to know a law of nature is to know something that remains constant when you pass from series to series. If various mountains, or rather elevated places, agree in certain respects with regard to the air pressure present at those levels, then I am in possession of a certain law relating to the connection between elevation and atmospheric pressure. But just as the sea level enables me to define this law, so the facts about the pressure at various places on the earth's surface might themselves be used to establish something equivalent to a level amongst all these places. If a great number of places on the earth's surface [18] agree at a given time in having the same pressure of atmosphere, these various places constitute a system of facts which stand in symmetrical transitive relations in so far as the pressure at any one place is equal to the pressure at another place. As soon as I ask what is the cause of this equality of pressure, and find its cause in elevation or in any other constant physical condition, I then inevitably set the places which have the same atmospheric pressure on a level, while at the same time I place in a series various places which have different atmospheric pressure. Consequently, in my mind I arrange all the places on the earth's surface with reference to atmospheric pressure in what may be called a series of levels. All those places which have the same atmospheric pressure being at a given moment on the same level, different places which have different atmospheric pressures being upon different levels, and all these levels themselves forming series.

To change the instance once more, if we return to the socially successful man in the various societies, he may define his laws to himself by defining certain classes of social situations, which in a certain respect are all alike, or are on the same level. Thus there is something that one does in greeting, there is something that one does in making or in returning compliments, in accepting or declining invitations, etc. All greetings, compliments, acceptances, acts of declining, and so on, are social facts, each state of each are on the same level. And now if one can define one's social laws one defines how one's acts would alter their character as the social levels in question change. Only in case one could define the possibilities of changing situations, the ways in which different types of invitations ought to be accepted or rejected, only in so far as one [19] could do this would one have significant laws of social behavior, and as soon as one did, these various social levels would be again arranged in a series.

In view of these considerations we may get a general view of what happens in all our scientific thinking if we think of the matter in this way. Suppose all the changes that occur in the world to be arranged in a system, so that all the changes were formed into series in so far as any set of changes gave good reason for regarding them as members of a single series, while all these series were grouped in some way side by side as lines may be grouped in a diagram. Suppose then various cross lines such as levels are drawn from part to part of this diagram, and suppose hereby one correlates various parts of one series with various parts of another, then the result is that one gets the sort of insight into the laws of things that we human beings are constantly endeavoring to get.

My result so far is that since our knowledge of law depends upon this endless arranging, grouping, and regrouping of series and systems of series of transformations, our whole knowledge of the laws of nature consists in such serial arrangement, and such observation of what remains unchanged through varying conditions, and in consequence includes all sorts of artificial rearrangements of our experience. Our science is not a mere report of facts of experience. It is a rearrangement of these facts. It is not an account of how the world would look if one took no interest in these series that we so carefully consider and arrange. It is an account of the world considered with reference to such series as we find it interesting to construct. If the facts decline to permit any such construction we should indeed be helpless. That the world permits us to make such constructions is [20] if you like, something of great value in estimating what kind of world this really is. But on the other hand, our own constructive activity constitutes by far the larger portion of our scientific insight into the laws of things. When we arrange events in series we are never merely at the mercy of the order in which the events happen to occur in time. Even the historian artificially rearranges his facts, establishing those levels and distinguishing those series which interest him. In any series there must be something that remains unchanged throughout the series, but this something must be something that so appeals to us, to our human interest, to our practical concern in the fact that our attention is attracted by it. It is essentially *we* who remain unchanged in a given case when we observe a given system of facts. Our interest, our experience, our attitude, our valuation of certain facts, remain in such wise unchanged that we are able to appreciate these facts as forming a single series. Leave our interest our way of experiencing, our attitude, our fashion of construction out of account, and you have no sense left in which the facts can be said to form a series. On the other hand, when we correlate many series together, we do so in a distinctly artificial fashion. The levels that interest us are again determined by certain unchanging features which we find in various series as we pass from one to another.

These unchanging features exist with such reference to our own interest, to our own attitude, to our own attentive deeds that if you abstract from our part in the process it is quite impossible to say what the facts alone by themselves would determine. The kind of significance which all this arrangement of facts in series, and of this study of the various levels has, our social instance has plainly shown us. The man to whom social situations form series exemplifying various [21] laws, because they are capable of being considered with reference to certain constant social interests and fashions of behavior, deals with the social world in a practical way and arranges his series and levels for the sake of success. The official coaling [sic] the squadron has to deal with facts, which indeed in one sense are stubborn enough, while in another sense they fall into these series and exemplify these laws because he approaches them with a certain human interest and because he wishes to accomplish a certain task. Abstract from the kind of success that we are seeking in our dealings with nature and you have no aspect of nature left which defines any precise system of laws. Beings interested in arranging facts otherwise, beings whose attention was attracted by other changeless aspects of their experience than those which interest us, might come to a different view of the laws of nature, as beings with wholly different sense organs from ours would reach. It is true that when we deal with nature we are not in the least privileged to be capricious. But on the other hand, we are not in the least able to abstract from our own forms of thoughtful activity and to define what the natural world would be if we had a wholly different type of conception.

We have now reviewed some of the fundamental conceptions of science, and we have found that they are distinctly human in their character as they are empirical in their illustrations. Nature permits you to conceive her fact in certain systems, but what these systems are your own interest as a thinker determines. Nature does not say to you, "Such I am and such I would be if you were not conceiving me." Nature says "Thus I present myself to your sense and thus I permit you if you are laborious enough and patient enough to succeed in rearranging my facts so that your industrial art makes its [22] triumphs and so that your scientific theories assume their well known types." The vastest generalizations that we make concerning nature conform to this principle as well as do the simplest. The great principles of the theory of energy of which our modern scientific inquirers rightly make so much are simply principles relating to series and to levels,-- the series and the levels in question being such as have no meaning apart from certain very human processes of description, definition, and rearrangement of facts. It is possible, experience permits it, that we should correlate certain physical facts with quantities of energy, with a series of facts known as our own numbers, so that as we say we measure anything, and assign a certain number to a certain energy, as naming the quantity of the energy present under the given conditions. All the objects that we deal with in such a case are not simple experiences, but highly transformed explanations of things which experience permits us to define. After we have once thus arranged in series quantities of energy, we are able to define a certain law or pair of laws which may be said to govern the transformations of energy into physical systems known to us. The first of this pair of laws is the socalled [sic] principle of the conservation of energy. It is the principle that all the transformations of energy of a closed system constitute a level. The other principle is that energy tends to pass from higher to lower levels of intensity, within the same system. From this point of view the series of transformations of energy, is a series whose states are not upon the same level, but run through a certain irreversible series of changes, energy in this sense passing from higher to lower levels [23] of intensity. The whole theory of energy is thus a theory of relating to series and levels conceived energy. The human interests which led to these conceptions are perfectly obvious to anyone who will consider the conceptions concerned. When we say that the facts of the world are of this type we mean the facts as conceivable and as capable of arrangement by us. The conception and the arrangement in question are very highly artificial. The arrangements in question are made by us for purposes which are not only theoretical but practical. By making such arrangements we guide our conduct. We understand nature in this field, precisely in so far as we can guide our industrial conduct, and our conduct as scientific conceivers of phenomena with actual success. When nature permits us to raise potatoes I do not suppose that it is a very notable revelation of the absolute nature of things, as they would be apart from human ingenuity, to say that the world permits potato raising. When we say that the principle of the conservation of energy is the truth, we substantially mean that man makes a success, industrial and theoretical, on the basis of arranging his conceptions of physical phenomena in a series and upon the levels which the theory of energy employs. We have no right to say that this fact reveals to us what the world would be by itself if men were not here conceiving it. The success of our arts and the success of our sciences is of substantially the same philosophical significance. Nature permits both if we work hard enough. Neither success is by itself sufficient to decide for us what the ultimate nature of things may be.